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Previously Unknown Physical Formulas which Hold in a Hydrogen Atom and are Derived without Using Quantum Mechanics

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Abstract

It is thought that quantum mechanics is the physical science describing the behavior of the electron in the micro world, e.g., inside a hydrogen atom. However, the author has previously derived the energy-momentum relationship which holds inside a hydrogen atom. This paper uses that relationship to investigate the relationships between physical quantities which hold in a hydrogen atom. In this paper, formulas are derived which hold in the micro world and make more accurate predictions than the classical quantum theory. This paper concludes that quantum mechanics is not the only theory enabling investigation of the micro world.

Keywords: Einstein's energy-momentum relationship, Dirac's relativistic wave equation, Relativistic energy, Fine structure constant

1. Introduction

It is thought that quantum mechanics is the physical science describing the behavior of the electron in the micro world, e.g., inside a hydrogen atom. There is no difference of opinion on that point. However, is it impossible to investigate the micro world without quantum mechanics?

The author has previously derived the energy-momentum relationship which holds inside a hydrogen atom. This paper uses that relationship to investigate the relationships between physical quantities which hold in a hydrogen atom. A comparison is also made between the values of physical quantities predicted by this paper, and the values predicted by the classical quantum theory developed by Bohr. In this paper, formulas are derived which hold in the micro world and make more accurate predictions than the classical quantum theory.

2. Results Obtained Prior to this Paper

Letting m_0c^2 be the rest mass energy and $\boldsymbol{p}$ the momentum of an object or a particle existing in free space, Einstein's energy-momentum relationship is given by the following equation:

$$\left(mc^2\right)^2 = \boldsymbol{p}^2c^2 + \left(m_0c^2\right)^2. \tag{1}$$

Here, mc^2 is the relativistic energy.

In contrast, the author has derived the following relationship for a bound electron in a hydrogen atom, which must take into account the Coulomb potential (Suto, 2011):

$$E_{\mathrm{re},n}^2 + \boldsymbol{p}_n^2c^2 = \left(m_{\mathrm{e}}c^2\right)^2, \qquad n = 1, 2, \cdots. \tag{2}$$

Here, $E_{\mathrm{re},n}$ is the following relativistic energy of the electron, and the electron's energy is described on an absolute scale.

$$E_{\mathrm{re},n} = m_{\mathrm{e}}c^2 + E_n = m_{\mathrm{re},n}c^2, \qquad E_n < 0. \tag{3}$$

Here, $m_{\mathrm{re},n}$ is the relativistic mass of the electron. In Equation (3), E_n is the total mechanical energy of a hydrogen atom. The equation derived from classical quantum theory is following:

$$E_n = -\frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{m_e e^4}{\hbar^2} \cdot \frac{1}{n^2} = -\frac{\alpha^2 m_e c^2}{2n^2}, \quad n = 1, 2, \cdots. \tag{4}$$

Here, α is the fine structure constant as follows.

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c}. \tag{5}$$

The author presented the following equation as an equation indicating the relationship between the rest mass energy and potential energy of the electron (Suto, 2009).

$$V(r) = -\Delta m_e c^2. \tag{6}$$

According to this equation, the potential energy of a bound electron in a hydrogen atom is equal to the reduction in rest mass energy of that electron.

There is a lower limit to potential energy, and the range which energy can assume is as follows.

$$-m_e c^2 \le V(r) < 0. \tag{7}$$

Also, the following constraint holds regarding the relativistic energy E_{re} of the electron due to Equations (3) and (4) (here, the discussion is limited to the ordinary energy levels of the atom).

$$\frac{1}{2} m_e c^2 \le E_{re,n} < m_e c^2. \tag{8}$$

The logic used when deriving Equation (2) can also be applied in the region of Equation (8). Therefore, the author previously pointed out that there is an $n = 0$ energy level in a hydrogen atom, but that is a mistake (Suto, 2014a). Here, that will be corrected.

Incidentally, it is known that the following formula can be derived from Equation (1).

$$E = m_0 c^2 \left(1 - \frac{v^2}{c^2}\right)^{-1/2}. \tag{9}$$

If the same logic is applied to Equation (2), then the following formula can be derived. (See Appendix A)

$$E_{re} = m_{re} c^2 = m_e c^2 \left(1 + \frac{v^2}{c^2}\right)^{-1/2}. \tag{10}$$

Within a hydrogen atom, the mass of the electron decreases when the kinetic energy of the electron increases. That is, $m_0 < m$ in Equation (1), but in Equation (2) $m_{re,n} < m_e$.

Next, when a Taylor expansion is performed on Equation (10),

$$E_{re} \approx m_e c^2 \left(1 - \frac{v^2}{2c^2} + \frac{3v^4}{8c^4}\right). \tag{11}$$

In the theory of Dirac, the energy levels of the hydrogen atom can be expressed with the following equation (Schiff, 1968).

$$E_n = m_e c^2 \left[1 - \frac{\alpha^2}{2n^2} - \frac{\alpha^4}{2n^4}\left(\frac{n}{|k|} - \frac{3}{4}\right)\right]. \tag{12}$$

Thus this paper makes the following assumption, based on a comparison of Equations (11) and (12).

$$\frac{v_n}{c} = \frac{\alpha}{n}, \qquad n = 1, 2, \cdots. \tag{13}$$

However, the velocity is taken to be the average velocity of the electron, in accordance quantum mechanics. Here, v on the left side was set to v_n. There are also other reasons for assuming Equation (13). (See Appendix B)

In the classical quantum theory, the following quantum condition of Bohr plays an important role.

$$p_n \cdot 2\pi r_n = 2\pi n\hbar, \qquad n = 1, 2, \cdots. \tag{14}$$

However, in this paper Equation (13) is assumed instead of Equation (14). When this is done, the following $E_{\mathrm{re},n}$ and E_n can be derived from Equations. (10) and (13).

$$E_{\mathrm{re},n} = m_{\mathrm{re},n}c^2 = m_{\mathrm{e}}c^2\left(\frac{n^2}{n^2+\alpha^2}\right)^{1/2}, \qquad n = 1, 2, \cdots. \tag{15}$$

$$E_n = m_{\mathrm{e}}c^2\left[\left(\frac{n^2}{n^2+\alpha^2}\right)^{1/2} - 1\right]. \tag{16}$$

E_n of Equation (12) and $E_{\mathrm{re},n}$ of Equation (15) define an absolute quantity, which includes the electron's rest mass energy. Whereas E_n in Equations (4) and (16) express the reduction in rest mass energy of the electron.

Also, if Equation (15) is substituted into Equation (2),

$$p_n = m_{\mathrm{e}}c\left(\frac{\alpha^2}{n^2+\alpha^2}\right)^{1/2}. \tag{17}$$

From this, it is evident that Equation (2) has a structure like the following.

$$\left[m_{\mathrm{e}}c\left(\frac{n^2}{n^2+\alpha^2}\right)^{1/2}\right]^2 + \left[m_{\mathrm{e}}c\left(\frac{\alpha^2}{n^2+\alpha^2}\right)^{1/2}\right]^2 = \left(m_{\mathrm{e}}c\right)^2. \tag{18}$$

Equation (18) can also be written as follows.

$$\left(m_{\mathrm{re},n}c\right)^2 + \left(\frac{\alpha}{n}m_{\mathrm{re},n}c\right)^2 = \left(m_{\mathrm{e}}c\right)^2. \tag{19}$$

Incidentally, the energy of the electron in a hydrogen atom can be given not only by Equation (4) but also by the following formula.

$$E = -\frac{1}{2}\frac{e^2}{4\pi\varepsilon_0 r}. \tag{20}$$

Here, if $-m_{\mathrm{e}}c^2$ is substituted for E in Equation (20), then the r where $E_{\mathrm{re}} = 0$ is

$$r = \frac{r_{\mathrm{e}}}{2}. \tag{21}$$

Here, r_{e} is the classical electron radius as follows.

$$r_{\mathrm{e}} = \frac{e^2}{4\pi\varepsilon_0 m_{\mathrm{e}}c^2}. \tag{22}$$

3. Formulas containing the Fine Structure Constant

The following equation can be obtained from Equation (15).

$$\frac{m_{\mathrm{e}}}{m_{\mathrm{re},n}} = \left(\frac{n^2+\alpha^2}{n^2}\right)^{1/2}. \tag{23}$$

If $n = 1$ here,

$$\left(\frac{m_{\mathrm{e}}^2 - m_{\mathrm{re},1}^2}{m_{\mathrm{re},1}^2}\right)^{1/2} = \alpha. \tag{24}$$

Incidentally, Equation (20) can also be written as follows.

$$E_n = -\frac{1}{2}\frac{r_e m_e c^2}{r_n} = -m_e c^2\left(\frac{r_e/2}{r_n}\right). \tag{25}$$

Also, the following formula for energy and momentum can be obtained from Equations (2) and (25).

$$E_{\mathrm{re},n} = m_e c^2 + E_n = m_e c^2\left(1-\frac{r_e/2}{r_n}\right). \tag{26}$$

$$p_n = m_e c\left[1-\left(1-\frac{r_e/2}{r_n}\right)^2\right]^{1/2}. \tag{27}$$

Furthermore, it is evident from Equations (23) and (26) that the following relationship holds.

$$\frac{E_{\mathrm{re},n}}{m_e c^2} = \left(\frac{n^2}{n^2+\alpha^2}\right)^{1/2} = \frac{m_{\mathrm{re},n}}{m_e} = \frac{r_n - r_e/2}{r_n}. \tag{28}$$

Here, $(r_n - r_e/2)$ is distance in the $0 \le E_{\mathrm{re}}$ region within the orbital radius r_n (In contrast, $r_e/2$ is distance in the $E_{\mathrm{re}} \le 0$ region).

When Equations (24) and (28) are taken into account, the following formula containing the fine structure constant is obtained.

$$\left[\left(\frac{r_1}{r_1 - r_e/2}\right)^2 - 1\right]^{1/2} = \alpha. \tag{29}$$

Also, the following formula is obtained from Equation (28).

$$r_n = \frac{r_e}{2}\frac{m_e}{m_e - m_{\mathrm{re},n}} = \frac{r_e}{2}\left(1-\frac{m_{\mathrm{re},n}}{m_e}\right)^{-1}. \tag{30}$$

In quantum mechanics, *r* is an average value not a definitive value, and this paper follows that principle.

Now, if a Taylor expansion is performed on the right side of Equation (15),

$$\left(\frac{n^2}{n^2+\alpha^2}\right)^{1/2} = \left(1+\frac{\alpha^2}{n^2}\right)^{-1/2} \approx 1-\frac{\alpha^2}{2n^2}+\frac{3\alpha^4}{8n^4}. \tag{31}$$

This yields,

$$1-\left(\frac{n^2}{n^2+\alpha^2}\right)^{1/2} \approx 1-\left(1-\frac{\alpha^2}{2n^2}+\frac{3\alpha^4}{8n^4}\right) \approx \frac{\alpha^2}{2n^2}. \tag{32}$$

Based on this result, Equation (28),

$$\frac{r_e/2}{r_n} = 1-\frac{m_{\mathrm{re},n}}{m_e} = -\frac{E_n}{m_e c^2} = 1-\left(\frac{n^2}{n^2+\alpha^2}\right)^{1/2} \approx \frac{\alpha^2}{2n^2}. \tag{33}$$

From this,

$$E_n \approx -\frac{\alpha^2}{2n^2} m_e c^2. \tag{34}$$

In the end, it is evident that Equation (4) is an approximation.

Also, if r_n is found from Equation (33),

$$r_n = \frac{r_e}{2}\left[1-\left(\frac{n^2}{n^2+\alpha^2}\right)^{1/2}\right]^{-1}. \tag{35}$$

4. Formulas determined to be Approximations

This section discusses the orbital radii and energy levels of a hydrogen atom derived by Bohr.

1) Orbital radii of Bohr

Dirac pointed out that there is a negative solution to Equation (1). Adopting the same viewpoint, there is a negative solution to Equation (2). To find the negative solution, it is necessary to create a quadratic equation for *r*. Thus, from Equation (28),

$$\left(\frac{r_n - r_e/2}{r_n}\right)^2 = \frac{n^2}{n^2+\alpha^2}. \tag{36}$$

From this, the following quadratic equation is obtained.

$$r_n^2 - \left(\frac{n^2+\alpha^2}{\alpha^2}\right) r_e r_n + \left(\frac{n^2+\alpha^2}{\alpha^2}\right)\frac{r_e^2}{4} = 0. \tag{37}$$

If this equation is solved for r_n,

$$r_n = \frac{1}{2}\left\{\left(\frac{n^2+\alpha^2}{\alpha^2}\right) r_e \pm \left[\left(\frac{n^2+\alpha^2}{\alpha^2}\right)^2 r_e^2 - 4\left(\frac{n^2+\alpha^2}{\alpha^2}\right)\frac{r_e^2}{4}\right]^{1/2}\right\}. \tag{38}$$

Rearranging this equation,

$$r_n = \frac{1}{2}\left(1+\frac{n^2}{\alpha^2}\right) r_e \pm \frac{1}{2}\left(1+\frac{n^2}{\alpha^2}\right) r_e \left[1-\left(1+\frac{n^2}{\alpha^2}\right)^{-1}\right]^{1/2} \tag{39a}$$

$$= \frac{r_e}{2}\left(1+\frac{n^2}{\alpha^2}\right)\left[1 \pm \left(1+\frac{\alpha^2}{n^2}\right)^{-1/2}\right]. \tag{39b}$$

To begin, the positive solution is found first. (The positive solution is the solution found by Bohr.) Letting r_n^+ be this solution, and performing Taylor expansion of Equation (39b),

$$r_n^+ \approx \frac{r_e}{2}\left(1+\frac{n^2}{\alpha^2}\right)\left[1+\left(1-\frac{\alpha^2}{2n^2}+\frac{3\alpha^4}{8n^4}\right)\right] \tag{40a}$$

$$\approx \frac{3r_e}{4} + \frac{r_e n^2}{\alpha^2} = \frac{3r_e}{4} + a_B n^2. \tag{40b}$$

In contrast, the radii r_n found by Bohr are given by the following equation.

$$r_n = a_B n^2 = 4\pi\varepsilon_0 \frac{\hbar^2}{m_e e^2}\cdot n^2 = \frac{r_e}{\alpha^2} n^2, \qquad n = 1, 2, \cdots. \tag{41}$$

Here, a_B is the Bohr radius. If Equations. (40) and (41) are compared, it is evident that Equation (41) is an approximation.

Next, the negative solution r_n^- of Equation (39b),

$$r_n^- \approx \frac{r_e}{2}\left(1+\frac{n^2}{\alpha^2}\right)\left[1-\left(1-\frac{\alpha^2}{2n^2}+\frac{3\alpha^4}{8n^4}\right)\right] \tag{42a}$$

$$\approx \frac{r_{\mathrm{e}}}{4} + \frac{\alpha^2 r_{\mathrm{e}}}{16n^2}. \tag{42b}$$

Since r^- converges to $r_{\mathrm{e}}/4$, $r_{\mathrm{e}}/4$ can be regarded as the radius of the atomic nucleus of a hydrogen atom (i.e., the proton). Here,

$$r_1^- \left(\frac{r_{\mathrm{e}}}{4}\right)^{-1} \approx 1 + \frac{\alpha^2}{4} = 1.0000133. \tag{43}$$

From this, it is evident that the negative orbital is located near the atomic nucleus. The author has pointed out that, if electron exist in this orbital, they will be a candidate for the dark matter whose real nature is currently unknown (Suto, 2017).

2) Energy levels

There are also positive and negative solutions for $E_{\mathrm{re},n}$ in Equation (2). Here, the ordinary, known energies of a hydrogen atom are expressed as $E_{\mathrm{re},n}^+$, E_n^+. Also, the negative energies are expressed as $E_{\mathrm{re},n}^-$, E_n^-.

When this is done, the formulas for positive energies are as follows.

$$E_{\mathrm{re},n}^+ = m_{\mathrm{e}}c^2\left(1+\frac{\alpha^2}{n^2}\right)^{-1/2} \approx m_{\mathrm{e}}c^2 - \frac{\alpha^2}{2n^2}m_{\mathrm{e}}c^2, \qquad n = 1,2,\cdots. \tag{44}$$

$$E_n^+ = m_{\mathrm{e}}c^2\left[\left(\frac{n^2}{n^2+\alpha^2}\right)^{1/2} - 1\right] \approx -\frac{\alpha^2}{2n^2}m_{\mathrm{e}}c^2 = -\frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{m_{\mathrm{e}}e^4}{\hbar^2}\cdot\frac{1}{n^2}. \tag{45}$$

In contrast, the formulas for the negative solutions are as follows.

$$E_{\mathrm{re},n}^- = -m_{\mathrm{e}}c^2\left(\frac{n^2}{n^2+\alpha^2}\right)^{1/2} \approx -m_{\mathrm{e}}c^2 + \frac{\alpha^2}{2n^2}m_{\mathrm{e}}c^2, \qquad n = 1,2,\cdots. \tag{46}$$

$$E_n^- = -m_{\mathrm{e}}c^2\left[\left(\frac{n^2}{n^2+\alpha^2}\right)^{1/2} + 1\right] \approx -2m_{\mathrm{e}}c^2 + \frac{\alpha^2}{2n^2}m_{\mathrm{e}}c^2. \tag{47}$$

The following compares energies when n=1.

Value predicted by Bohr Equation (4):

$$E_{\mathrm{B},1} = -13.60569 \text{ eV}. \tag{48a}$$

Value predicted by Dirac Equation (12):

$$E_{\mathrm{D},1} = -13.60514919 \text{ eV}. \tag{48b}$$

Value predicated by this paper Equation (16):

$$E_1 = -13.60514921 \text{ eV}. \tag{48c}$$

The predictions of Dirac and this paper agree to the sixth digit after the decimal point.

Incidentally, the Dirac's relativistic wave equation can be written as follows

$$i\hbar\frac{\partial}{\partial t}\psi = \left(-i\hbar c\alpha_j\nabla_j + \beta m_{\mathrm{e}}c^2\right)\psi. \tag{49}$$

The following form can be used as a matrix with four rows and four columns.

$$\alpha_j = \begin{pmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}. \tag{50}$$

Here, the following Pauli spin matrices and the unit matrix are used.

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{51}$$

In contrast, the author has derived the following equation (Suto, 2014b).

$$i\hbar \frac{\partial}{\partial t}\psi = \left(-i\hbar c\alpha_j' \nabla_j + \beta m_e c^2\right)\psi. \tag{52}$$

Where,

$$\alpha_j' = \begin{pmatrix} 0 & i\sigma_j \\ i\sigma_j & 0 \end{pmatrix}. \tag{53}$$

Equation (52) must be solved in order to elucidate electron spin, or more complex phenomena.

5. Discussion

1) The radius r where $E_{re} = 0$ is $r_e / 2$ due to Equation (21). Dirac predicted that the vacuum energy E_{re} satisfies the relation $E_{re} < -m_e c^2$, but actually $E_{re} = 0$ is the energy of the virtual electron-positron pair which make up the vacuum (see Fig. 1). Also, in quantum mechanics it is thought that atoms are more stable when electron energy is lower, but actually that is not the case. They are more stable as energy approaches closer to $E_{re} = 0$. The relationship between r and E_{re} is summarized as follows.

$$r_e \le r_n^+ \;\leftrightarrow\; \frac{1}{2} m_e c^2 \le E_{re,n}^+ < m_e c^2. \tag{54a}$$

$$\frac{r_e}{4} < r_n^- \le \frac{r_e}{3} \;\leftrightarrow\; -m_e c^2 < E_{re,n}^- \le -\frac{1}{2} m_e c^2. \tag{54b}$$

$$\frac{r_e}{3} \le r \le r_e \;\leftrightarrow\; -\frac{1}{2} m_e c^2 \le E_{re} \le \frac{1}{2} m_e c^2. \tag{54c}$$

$$r = \frac{r_e}{2} \leftrightarrow E_{re} = 0. \tag{54d}$$

Equation (2) is applicable in the ranges of Equations (54a) and (54b). Equation (54a) is the region where an ordinary hydrogen atom electron exists. Also, in Equation (54b), there is a system of an electron with negative mass and a proton with positive mass. Also, as is clear from Equation (8), the electron cannot penetrate into the region of Equation (54c). Therefore, E_{re} in Equation (54c) is likely not the energy of the electron, but rather the energy of a virtual electron-positron pair making up the vacuum. However, the energy of a virtual particle pair is twice the energy of the virtual electron, and thus Equation (54c) can be rewritten as follows.

$$\frac{r_e}{3} \le r \le r_e \;\leftrightarrow\; -m_e c^2 \le E_{vp} \le m_e c^2. \tag{55}$$

There is no electron energy level in the region of Equation (54c), and thus the subscript n is omitted. Also the subscript vp on the energy indicates a virtual particle pair.

Incidentally, potential energy does not exist in the region of Equation (54c). Therefore, if it is assumed that the energy-momentum relation holds for virtual particle pairs too, then that can be obtained by setting m_0 equal to zero in Equation (1). That is,

$$E_{vp} = m_{vp} c^2 = p_{vp} c. \tag{56}$$

Here, $m_{re,vp}$ indicates the mass of the virtual particle pair, and p_{vp} indicates the momentum of the virtual particle pair.

Dirac regarded the energy region of the vacuum to be as follows.

$$E_{re} < -m_e c^2. \tag{57}$$

In this paper, in contrast, the energy region of the vacuum in a hydrogen atom is predicted to be Equation (55).

2) The uncertainty principle is thought to be what guarantees the stability of the atom. According to the uncertainty principle, it is forbidden for the electron to approach the atomic nucleus $(r \to r_e / 4)$ and for the momentum to approach zero $(p \to 0)$. However, the situation is different in the following domain.

$$\frac{r_{\mathrm{e}}}{2} < r < r_{\mathrm{e}} \;\; \leftrightarrow \;\; 0 < E_{\mathrm{vp}} < m_{\mathrm{e}}c^2. \tag{58}$$

In this region, the momentum approaches zero as the virtual particle pair approaches $r_{\mathrm{e}}/2$. That is,

$$r \to \frac{r_{\mathrm{e}}}{2}, \quad p \to 0. \tag{59}$$

Also, taking an electron as an example, if the electron approaches the atomic nucleus in the region of Equation (54b), then the momentum approaches zero (this is clear from Equation (2)). That is,

$$r \to \frac{r_{\mathrm{e}}}{4}, \quad p \to 0. \tag{60}$$

From this it is evident that the uncertainty principle is not a universal principle, and there are limits on its application.

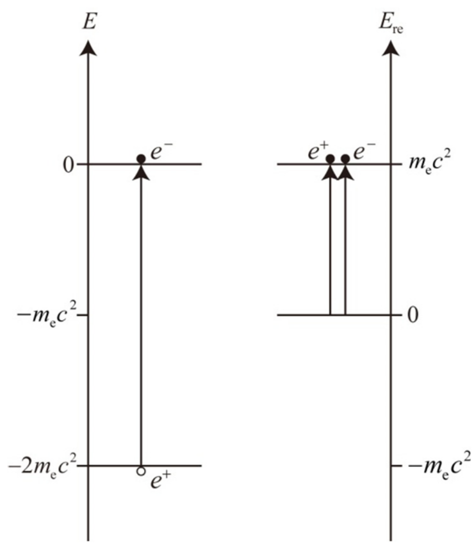

Fig. 1a Fig. 1b

Figure 1. Differences between Dirac's hole theory and the interpration in this paper

In Dirac's theory, when the γ-ray gives all of its energy to the virtual particles ($E = -2m_{\mathrm{e}}c^2$, i.e., $E_{\mathrm{re}} = -m_{\mathrm{e}}c^2$) comprising the vacuum around the atomic nucleus, a virtual particle acquires rest mass, and is emitted as an electron into free space, while the hole opened in the vacuum is the positron (Fig.1a).

In the author's interpretation, on the other hand, an electron-positron pair is created because a γ-ray with an energy of 1.022 MeV gives rest mass to a virtual electron-positron pair at the position $r = r_{\mathrm{e}}/2$ (Fig.1b). Arrows show the change in particle energy. The end point of an arrow does not indicate the position where the particle was produced.

6. Conclusion

In this paper, the departure point is Equation (2), and from there the relationships between physical quantities holding in a hydrogen atom were clarified as far as possible. The following assumption and relationship were powerful at that time.

$$\frac{v_n}{c} = \frac{\alpha}{n}. \tag{13}$$

$$\frac{E_{\mathrm{re},n}}{m_{\mathrm{e}}c^2} = \left(\frac{n^2}{n^2+\alpha^2}\right)^{1/2} = \frac{m_{\mathrm{re},n}}{m_{\mathrm{e}}} = \frac{r_n - r_{\mathrm{e}}/2}{r_n}. \tag{28}$$

Also, in this paper, the following formulas were derived with higher precision than Bohr's Formula (4) for energy levels.

$$E_{\mathrm{re},n} = \pm m_{\mathrm{e}} c^2 \left(\frac{n^2}{n^2 + \alpha^2} \right)^{1/2}. \tag{15}$$

$$E_{\mathrm{re},n} = \pm m_{\mathrm{e}} c^2 \left(\frac{r_n - r_{\mathrm{e}} / 2}{r_n} \right). \tag{26}$$

When Equation (28) is taken into account, it is evident that the right sides of the above two equations are all different expressions of $\pm m_{\mathrm{re},n} c^2$. Also, in this paper, the following formula with precision higher than Bohr's Formula (41) was derived for the orbital radius.

$$r_n = \frac{r_{\mathrm{e}}}{2} \left[1 - \left(\frac{n^2}{n^2 + \alpha^2} \right)^{1/2} \right]^{-1}, \qquad n = 1, 2, \cdots. \tag{35}$$

Just as with E_{re} there are the following two solutions for this orbital radius.

$$r_n^+ \approx \frac{3r_{\mathrm{e}}}{4} + \frac{r_{\mathrm{e}}}{\alpha^2} n^2 = \frac{3r_{\mathrm{e}}}{4} + a_{\mathrm{B}} n^2, \qquad n = 1, 2, \cdots. \tag{40b}$$

$$r_n^- \approx \frac{r_{\mathrm{e}}}{4} + \frac{\alpha^2 r_{\mathrm{e}}}{16 n^2}. \tag{42b}$$

Through this paper, it was possible to predict the values of physical quantities with greater accuracy than classical quantum theory by taking Equation (2) as a departure point. Due to the results obtained in this paper, the author believes that the correctness of Equation (2) has been demonstrated. The fact that the predictions of classical quantum theory are approximate values is already known. It is not the case that this paper has raised objections to quantum mechanics.

This paper concludes that quantum mechanics is not the only theory enabling investigation of the micro world.

Appendix A

We consider the energy of the electron inside the hydrogen atom by referring to the logic given in textbooks (French, 1968). If the velocity of the electron is set 0 in Equation (2), then the following equation of Einstein can be derived.

$$m_{\mathrm{e}} = \frac{E_{\mathrm{re}}}{c^2}. \tag{A1}$$

Also, in classical mechanics,

$$m_{\mathrm{e}} = \frac{p}{v}. \tag{A2}$$

From these two equations, we obtain:

$$cp = \frac{E_{\mathrm{re}} v}{c}. \tag{A3}$$

The physical quantities of the electron in the hydrogen atom take discrete values, and thus if the subscript n is attached to the physical quantities on both sides of Equation (A3), then

$$cp_n = \frac{E_{\mathrm{re},n} v_n}{c}. \tag{A4}$$

Substituting cp_n in Equation (A4) for Equation (2) here, simplifying and using the + value, we obtain:

$$E_{\mathrm{re},n} = m_{\mathrm{e}} c^2 \left(1 + \frac{v_n^2}{c^2} \right)^{-1/2}. \tag{A5}$$

Appendix B

In classical quantum theory, the hydrogen atom is explained using a model where an electron with negative charge rotates around a proton with positive charge due to the Coulomb attraction.

If the atomic nucleus is assumed to be at rest because it is heavy, then the electron (charge e, mass m_e) is regarded as rotating at a speed v along a circular orbit with radius r, centered on the nucleus.

The attraction which the electron receives from the proton is a central force, and the equation of motion can be expressed as follows.

$$\frac{e^2}{4\pi\varepsilon_0 r_n^2}=\frac{m_e v_n^2}{r_n}. \tag{B1}$$

Also, the quantum condition which Bohr assumed is following.

$$p_n\cdot 2\pi r_n = 2\pi n\hbar. \tag{B2}$$

From Equations (B1) and (B2),

$$\frac{v_n}{c}=\frac{\alpha}{n}. \tag{B3}$$

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