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Velocity addition laws which can coexist with the velocity addition law in the special theory of relativity

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Abstract: The only velocity addition law currently regarded as correct is that for the special theory of relativity. However, this paper concludes that there is a justification for the existence of velocity addition laws derived from the standpoint of Lorentz, who had doubts about the special theory of relativity. The velocity addition laws presented in this paper are not new, and are equations which are easy to understand in terms of common sense. Aside from the velocity addition law in the special theory of relativity, there are also velocity addition laws which match experimental results.
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Résumé: La seule loi d'addition des vitesses actuellement considérée comme correcte est celle de la théorie de la relativité restreinte. Toutefois, cet article conclut qu'il existe une justification de l'existence de lois d'addition des vitesses obtenues d'après le point de vue de Lorentz, qui avait des doutes quant à la théorie de la relativité restreinte. Les lois d'addition des vitesses présentées dans cet article ne sont pas nouvelles, et les équations qui les expriment sont faciles à comprendre en termes intuitifs. Outre la loi d'addition des vitesses de la théorie de la relativité restreinte, il existe donc des lois d'addition des vitesses en accord avec les résultats expérimentaux.

Key words: Special Theory of Relativity; Velocity Addition Law; Einstein; Lorentz.

I. INTRODUCTION

At the end of the 19th century, Michelson and Morley tried to detect the motion of the earth relative to the ether,[1] which was thought at that time to be in a state of "absolute rest." However, they were unable to detect the expected result.

To explain this experimental result, the physicists of the time developed the following interpretations:

(1) Michelson's interpretation: The reason why the expected result was not detected in the experiment is that the ether is stationary relative to the moving earth (i.e., the ether is moving together with the earth).
(2) Lorentz's interpretation: The reason why the ether could not be detected even though it exists is because the length of the earth contracts by $\sqrt{1-(v/c)^2}$ times in the direction of motion.[2]
(3) Einstein's interpretation: If the ether does not exist, then light propagates isotropically relative to the light source, and thus the experimental result is what one would expect.

Against this background, Einstein announced the special theory of relativity (STR). At that time, Einstein assumed the "principle of relativity" and the "principle of the constancy of the speed of light."

However, the latter includes the following two principles:

"Any ray of light moves in the "stationary" system of coordinates with the determined velocity c, whether the ray be emitted by a stationary or by a moving body."[3]

"Let a ray of light start at the "A time" t_A from A towards B, let it at the "B time" t_B be reflected at B in the direction of A, and arrive again at A at the "A time" t'_A.

In agreement with the experience, we further assume the quantity

$$\frac{2AB}{t'_A - t_A} = c,$$

to be a universal constant-the velocity of light in empty space."[4]

In this paper, we distinguish between the former principle as the "principle of the constancy of the speed of light I" and the latter principle as the "principle of the constancy of the speed of light II." (These are abbreviated below as "principle I" and "principle II.")

"Principle I" asserts that the speed of light does not depend on the speed of the light source. "Principle II" asserts that the speed of light calculated from the round-trip travel time is constant. Therefore, if "principle I" is taken into account, it is impossible to say with certainty based on "principle II" that the one-way speed of light is c.

Next, let us summarize the approaches of different physicists regarding the outward propagation of light.

"Isotropic propagation of light EM": Propagation of light predicted by Einstein and Michelson. ("E" stands for Einstein, and "M" for Michelson. This approach is abbreviated below as "propagation EM.") In this paper, a "classically stationary system" is defined as a system in which light propagates isotropically in the absolute sense.

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"Anisotropic propagation of light L": Propagation of light predicted by Lorentz. An observer in a stationary system applies "principle I" to the propagation of light in a moving system. At this time, an observer in the moving system determines that propagation of light in his own coordinate system is anisotropic. ("L" stands for Lorentz. This approach is abbreviated below as "propagation L.")

"Principle of the constancy of the speed of light E": Even if "propagation L" holds in a coordinate system, if the times of two clocks are synchronized within that coordinate system, then the propagation of light in this coordinate system will be isotropic in the relativistic sense. (This is not isotropic propagation in an *a priori*, absolute sense.) Also, the one-way speed of light will be measured as c.

As a result, all inertial systems will be equivalent, and the debate regarding identification of "propagation EM" and "propagation L" will come to an end. Einstein conceived of a new principle, the "principle of the constancy of the speed of light E," which integrates these two types of propagation, and introduced that principle to physics. (This approach is abbreviated below as "principle E.") In this paper, a "relativistically stationary system" is defined as a system in which light propagates isotropically in the relativistic sense.

Einstein himself did not develop the above categories and names. The categories were developed for this paper to clarify Einstein's approach.

According to the STR, it is impossible to experimentally distinguish "propagation EM" and "propagation L." Therefore, for Einstein, making such a distinction was meaningless.

However, in another paper,[5] the author has presented a thought experiment where it is possible to distinguish between coordinate systems where "propagation EM" and "propagation L" hold (see Appendix A). Therefore, this paper sees no problem in distinguishing "classically stationary systems" and "relativistically stationary systems." It will be confirmed that the stationary systems treated in the thought experiment in the following and subsequent sections are classically stationary systems.

II. RELATIVISTIC SYNCHRONIZATION OF TWO CLOCKS IN UNIFORM MOTION

Let there be a given stationary rigid rod of length L_0 as measured by a ruler which is stationary, and assume that the rod is placed along the positive direction of the stationary system x-axis.

Assume that clocks A and B of the same type are set up at points A and B on the rear and front end of this rod. Here clock A will be abbreviated as C_A, and clock B as C_B.

Suppose a ray of light is emitted in the direction of B from A at the time t_A of C_A, reaches and is reflected at B at time t_B of C_B, and then returns to A at time $t_{A'}$ of C_A. Einstein determined that if the following relationships hold, then the two clocks represent the same time by definition.[3]

$$t_B - t_A = t_{A'} - t_B, \tag{1}$$

$$\frac{1}{2}(t_A + t_{A'}) = t_B. \tag{2}$$

In this paper, let us adjust the time of C_B, and synchronize the times of C_A and C_B when the rod is stationary.

Also let us indicate C_B, whose time was adjusted while stationary at the beginning, as C_{B1}. (The 1 in B1 signifies that time was adjusted once. C_A is not adjusted, so its indication is not changed.)

Here, the times of C_A and C_B are synchronized because the author wishes to carry the discussion up to the time adjustment when resynchronizing clocks which have begun to move at a constant velocity.

Incidentally, Einstein's relation is also applied when synchronizing the times of two clocks in the coordinate system of a rod moving at a constant velocity.

Suppose a ray of light is emitted in the direction of B from A at the time t'_A of C_A, it reaches and is reflected at B at time t'_B of C_B, and then returns to A at time $t'_{A'}$ of C_A.

If the following relation holds between the times of the two clocks at this time, then the times of the two clocks are the same by definition.

$$t'_B - t'_A = t'_{A'} - t'_B, \tag{3}$$

$$\frac{1}{2}(t'_A + t'_{A'}) = t'_B. \tag{4}$$

Next, assume that the stationary rod has been accelerated, and has attained the constant velocity $\boldsymbol{v}$ (see Fig. 1).

Note that the velocities discussed in this paper will be assumed to be high velocities to which the STR is applicable.

In the acceleration stage during this interval, the times which elapse on C_A and C_{B1} are delayed compared with the time on the stationary clock.

Viewing from the stationary system, however, the times which elapse on C_A and C_{B1} are equal in this acceleration process.

If light propagates isotropically in the coordinate system where the rod was stationary at the beginning, then C_A and C_{B1} match absolutely. Also, Einstein's relation Eq. (3) holds between the times of C_A and C_{B1} in this stationary system, and thus relativistically the times of C_A and C_{B1} are synchronized. However, when this rod begins to move, the times of the two clocks remain absolutely synchronized, but it can no longer be said that they are relativistically synchronized. The reason for this is because, when the rod begins moving at a constant velocity, the relation in Eq. (3)

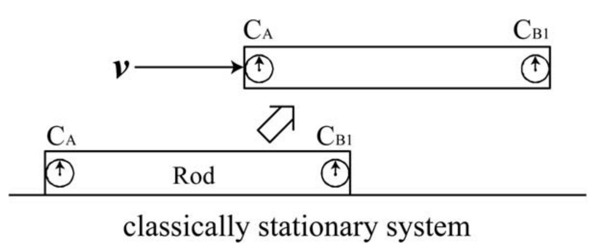

FIG. 1. First, the times of clocks, C_A and C_B, at both ends of a rod are synchronized while the rod is stationary in a classically stationary system. Then the rod begins moving at a constant velocity $\boldsymbol{v}$. (In this case C_B is indicated as C_{B1}.) The adjustment time of C_B when synchronizing the times of C_A and C_B cannot be predicted.

no longer holds between the times of C_A and C_{B1}. (This is clear on account of "principle I.")

Thus the time of C_{B1} must be readjusted so that the relation in Eq. (3) holds in the coordinate system of the rod moving at a constant velocity. (However, this time adjustment is the first and only adjustment in the STR.)

If this time adjustment is not made, C_A and C_{B1} will no longer be synchronized in the relativistic sense. Therefore, when the one-way speed of light is measured in that coordinate system, it will not be c (in other words, "principle E" will not hold).

This readjustment is not performed because time elapses differently between C_A and C_{B1}. It is also not performed because the clock in the moving system is running slower than the clock in the stationary system. This time adjustment is performed because the times of C_A and C_{B1} go out of relativistic synchronization due to the start of motion by the rod. (Here, the readjusted C_{B1} is indicated as C_{B2}).

In the end, this time adjustment is for setting the round-trip speed of light to c while maintaining "principle I."

III. ACTUAL ADJUSTMENT TIME FOR SYNCHRONIZING TWO CLOCKS

In Section II, we defined t'_A, t'_B, and $t'_{A'}$ in S'. Let us assume that t_A, t_B, and $t_{A'}$ in S correspond to these times. In this case, if the time needed for light to travel from A to B is measured with a clock in S, the result is $(t_B - t_A)$ seconds. According to the STR, when viewed from S, the rod contracts by $\sqrt{1-(v/c)^2}$ times in the direction of motion. In addition, when the velocity of light emitted from S' is seen from S, it is always constant regardless of the velocity of the light source (principle I), and thus $(t_B - t_A)$ is given by the following equation:

$$t_B - t_A = \frac{L_0\sqrt{1-(v/c)^2}}{c-v}\,(\mathrm{s}). \tag{5}$$

If the time needed for light to return from B to A is measured with the clock in S, and is taken to be $(t_{A'} - t_B)$ seconds, then

$$t_{A'} - t_B = \frac{L_0\sqrt{1-(v/c)^2}}{c+v}\,(\mathrm{s}). \tag{6}$$

However, the denominator on the right side of Eqs. (5) and (6) does not mean that the speed of light varies depend on the velocity of the light source.[5]

According to the STR, the relationship between $(t'_B - t'_A)$ and $(t_B - t_A)$ is

$$(t'_B - t'_A) = (t_B - t_A)\sqrt{1-(v/c)^2}. \tag{7}$$

If the right side of Eq. (5) is substituted for $(t_B - t_A)$ in Eq. (7),

$$t'_B - t'_A = \frac{L_0\left(\sqrt{1-(v/c)^2}\right)^2}{c-v} \tag{8a}$$

$$= \frac{L_0(c+v)}{c^2}\,(\mathrm{s}). \tag{8b}$$

If, in the same way, the time elapsed on a clock in S' while light returns from B to A $(t'_{A'} - t'_B)$ is measured by an observer in S,

$$t'_{A'} - t'_B = \frac{L_0(c-v)}{c^2}\,(\mathrm{s}). \tag{9}$$

If we set $t'_A = 0$ to simplify the equation, then the following value is obtained from Eqs. (8b) and (9):

$$\frac{1}{2}t'_{A'} = \frac{1}{2}\left[(t'_B - t'_A) + (t'_{A'} - t'_B)\right] \tag{10a}$$

$$= \frac{1}{2}\left[\frac{L_0(c+v)}{c^2} + \frac{L_0(c-v)}{c^2}\right] \tag{10b}$$

$$= \frac{L_0}{c}\,(\mathrm{s}). \tag{10c}$$

When light travels from A to B in S', an observer in S determines that $L_0(c+v)/c^2$ seconds have passed on the clock in S'. However, when light departing from A at $t'_A = 0$ arrives at B, the time on C_{B1} is L_0/c seconds. Since $L_0(c+v)/c^2 > L_0/c$, there is no contradiction if the time on C_{B1} is delayed compared with the time on C_A. Now, if we let $\Delta t'_{B1}$ be the adjustment time for C_{B1},

$$\Delta t'_{B1} = \frac{L_0(c+v)}{c^2} - \frac{L_0}{c} \tag{11a}$$

$$= \frac{L_0 v}{c^2}\,(\mathrm{s}). \tag{11b}$$

If an observer in S' delays the time on C_{B1} by $L_0 v/c^2$ seconds, then the relationship in Eq. (3) will hold in this coordinate system. As a result, "principle E" will hold in this coordinate system.

However, "principle E," which is specific to the STR, has the problem that it cannot survive as a principle unless the times of C_A and C_B are synchronized each time the velocity of the coordinate system changes.

IV. ADJUSTMENT OF CLOCK TIME NECESSARY FOR ENSURERING THE "PRINCIPLE OF THE CONSTANCY OF THE SPEED OF LIGHT E"

The velocity addition law in the STR is given by the following equation:

$$u = \frac{v+w}{1+\dfrac{vw}{c^2}}. \tag{12}$$

Here, v is the velocity of the moving system S' measured from the stationary system S, and w is the velocity of another moving system S'' measured from S'. Also, u is taken to be the velocity S'' measured by an observer in S. (The movement directions of S' and S'' are taken to be the positive direction of the x-axis of the stationary system. In this paper, the stationary system S is abbreviated S, the moving system S' is abbreviated S', and the moving system S'' is abbreviated S''.)

Now, consider the case where two rods are placed parallel with the x-axis in the stationary system. (The two rods will be distinguished as rod 1 and rod 2.) Here, the clocks at each end of rod 1 will be C_{1A} and C_{1B}, and the clocks at each end of rod 2 will be C_{2A} and C_{2B}. It is assumed that the times of C_{1A} and C_{1B}, as well as C_{2A} and C_{2B} are synchronized when the clocks are at rest. (Once their times have been adjusted, C_{1B} will be indicated as C_{1B1}, and C_{2B} will be indicated as C_{2B1}.)

Next, consider the case when rod 1 begins to move at the constant velocity $\boldsymbol{v}$, and at the same time rod 2 begins to move at the constant velocity $\boldsymbol{v}'$ in the positive direction of the x-axis of the stationary system. (However, it is assumed here that $v < v'$. Also, let the coordinate system of rod 1 be S'_1, and let the coordinate system of rod 2 be S'_2.) When these rods begin moving at constant velocity, the times of the clocks must be readjusted.(see Fig. 2) If the adjusted time of C_{1B1} is assumed to be $\Delta t'_{1B1}$, and the adjusted time of C_{2B1} is assumed to be $\Delta t'_{2B1}$, then

$$\Delta t'_{1B1} = \frac{Lv}{c^2}, \quad \Delta t'_{2B1} = \frac{Lv'}{c^2}. \tag{13}$$

Here, C_{1B1} after time adjustment the second time is indicated C_{1B2}, and similarly, C_{2B1} is indicated C_{2B2}.

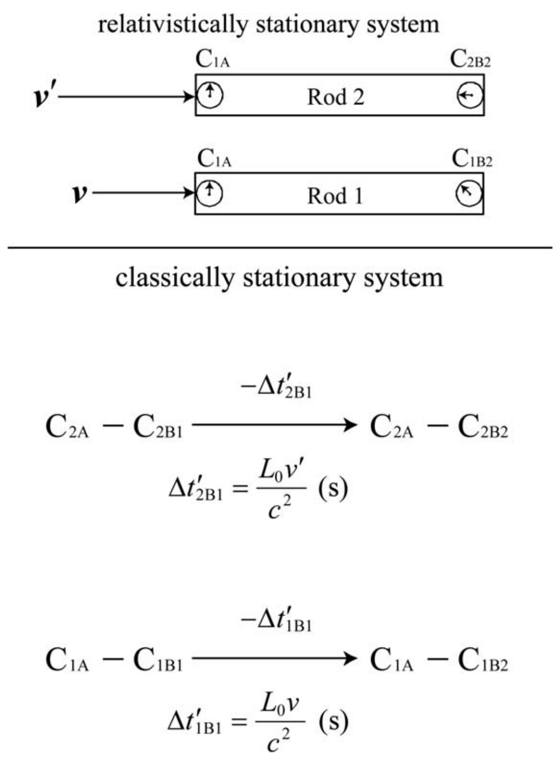

FIG. 2. The time adjustments are $\Delta t'_{1B1}$ and $\Delta t'_{2B1}$ for C_{1B1} and C_{2B1} which move at the constant velocities $\boldsymbol{v}$ and $\boldsymbol{v}'$ with respect to the classically station system. The coordinate systems of Rod 1 and Rod 2 acquire the status of relativistic stationary systems due to performing this time adjustment.

Now, the relativistic velocity $\boldsymbol{w}_r$ is measured using these adjusted clocks in the coordinate systems of the two rods. As a result, it becomes possible for an observer in the stationary system to apply the following equations as the velocity addition law in the STR

$$u = \frac{v + w_{1r}}{1 + \frac{vw_{1r}}{c^2}}, \quad u' = \frac{v' + w_{2r}}{1 + \frac{v'w_{2r}}{c^2}}. \tag{14}$$

Here, w_{1r} signifies the relativistic velocity measured in S'_1, and w_{2r} the relativistic velocity measured in S'_2. (The clocks used to measure $\boldsymbol{w}_{1r}$ are C_{1A} and C_{1B2}, and the clocks used to measure $\boldsymbol{w}_{2r}$ are C_{2A} and C_{2B2}.)

Next, consider the case where rod 1, moving at constant velocity $\boldsymbol{v}$, is accelerated until its velocity becomes $\boldsymbol{v}'$, and then it continues moving at a constant velocity (see Fig. 3).

In order to measure the relativistic velocity $\boldsymbol{w}_r$ in the coordinate system of the rod whose velocity was changed from $\boldsymbol{v}$ to $\boldsymbol{v}'$, the time of C_{1B2} must be readjusted so that C_{1A} and C_{1B2}, which were synchronized in S'_1, are synchronized in S'_2.

In this case, it is enough to set back the time of C_{1B2} by $(\Delta t'_{2B1} - \Delta t'_{1B1})$(s). This is the second adjustment since the rod started moving, and the third adjustment including that made when the rod was stationary. (After the time adjustment, C_{1B2} is indicated as C_{1B3}.)

When this situation is viewed by an observer in the stationary system, the time difference between C_{2A} and C_{2B2} is equal to the time difference between C_{1A} and C_{1B3}.

By making this time adjustment, it becomes possible to measure the relativistic velocity $\boldsymbol{w}_r$ in the coordinate system of rod 1, which has reached velocity $\boldsymbol{v}'$.

What the author wishes to point out as an unnatural time adjustment is this third time adjustment. When the rod

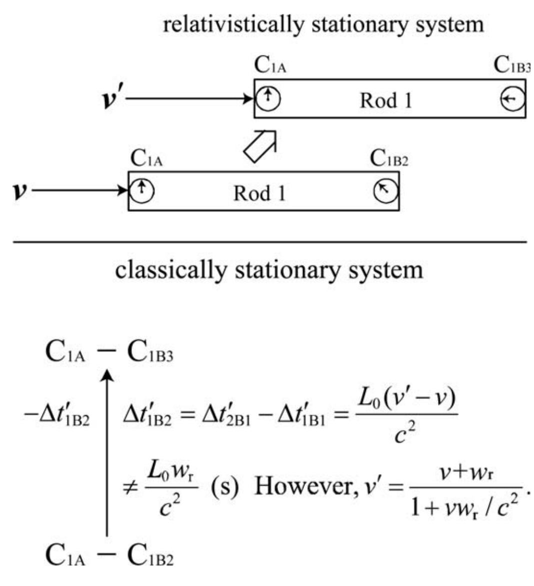

FIG. 3. If Rod 1 is accelerated and its velocity is changed from $\boldsymbol{v}$ to $\boldsymbol{v}'$, then the time of C_{1B2} must be delayed further by $\Delta t'_{1B2}$. If this time adjustment is not performed, it will be impossible for the coordinate system of the rod whose velocity was changed to maintain its status as a relativistic stationary system.

changes its velocity in the coordinate system of the rod, time adjustment of the clocks must be repeated. If this time adjustment is not performed in the moving system, then "principle E" will not hold in the moving system. In addition, an observer in the moving system will not be able to measure the relativistic velocity $\boldsymbol{w}_{\mathrm{r}}$ using the two clocks. As a result, it will be impossible for an observer in the stationary system to apply the velocity addition law of the STR.

Normally, with the STR, thought experiments have not been taken this far, and thus this problem did not arise.

Adjusting clocks is not a problem in a simple thought experiment, but actually doing this over and over would be a very troublesome task. Now, let w be the velocity of S'' measured using C_A and C_B, synchronized at the beginning when the rod was stationary. If this w is used, what sort of velocity addition law will an observer in the stationary system derive?

V. CLASSICAL VELOCITY ADDITION LAWS DERIVED FROM THE LORENTZ PERSPECTIVE

Now let us consider a situation where, in addition to C_{B1}, another clock C_{B2} of the same type as C_{B1} but with time set $L_0 v/c^2$ seconds different from C_{B1} is placed at the front end of the rod treated in the thought experiment in Section III.

If the time needed for an object, which moves from A in the movement direction of the rod starting at $t'_A = 0$, to reach B, is measured with C_{B1}, it is measured as t'_{B1}. If it is measured with C_{B2}, it is measured as t'_{B2}.

Here, if we let w be the velocity derived from t'_{B1}, and w_r be the relativistic velocity derived from t'_{B2}, then w and w_r are given by the following equations:

$$w = \frac{L_0}{t'_{B1}}, \quad w_r = \frac{L_0}{t'_{B2}}. \tag{15}$$

Taking into account Eq. (11b), there is the following relationship between t'_{B1} and t'_{B2}:

$$t'_{B1} = t'_{B2} + \frac{L_0 v}{c^2}. \tag{16}$$

From Eqs. (15) and (16), we can derive

$$\frac{L_0}{w} = \frac{L_0}{w_r} + \frac{L_0 v}{c^2}. \tag{17}$$

This leads to the following relationship between w and w_r:

$$w = \frac{w_r}{1 + v w_r/c^2}. \tag{18}$$

Incidentally, in this paper w_r is used instead of w in Eq. (12), and thus Eq. (12) becomes as follows:

$$u = \frac{v + w_r}{1 + v w_r/c^2}. \tag{19}$$

Also, Eq. (19) can be rewritten as follows:

$$u = \frac{v(c^2 + v w_r) + w_r c^2 - v^2 w_r}{c^2 + v w_r} \tag{20a}$$

$$= v + \frac{w_r(1 - v^2/c^2)}{1 + v w_r/c^2}. \tag{20b}$$

Here, if the right side of Eq. (18) is compared with the second term of Eq. (20b), then u becomes as follows:

$$u = v + w\left(1 - \frac{v^2}{c^2}\right). \tag{21}$$

This is the velocity addition law written using w in Eq. (18) in place of w_r in Eq. (19).

According to Einstein and Lorentz, the time which elapses in S' while a time of 1 s elapses in S is $\sqrt{1 - (v/c)^2}$ (s). Therefore, if an observer in S looks at an object which moves w (km) in S' while 1 s elapses in S', it will only move $w\sqrt{1 - (v/c)^2}$(km) while 1 s elapses in S. In addition, a rod which moves at a constant velocity $\boldsymbol{v}$ will contract $\sqrt{1 - (v/c)^2}$ times in the direction of motion.

Therefore, if the velocity of an object which moves w (km) in 1 s in S' is measured from S, the result will be $w(1 - v^2/c^2)$ (km/s).

If we set $w_r = c$ in Eq. (18), then w will be the speed of light measured using C_{B1}. If now we let this speed be w_c, then

$$w_c = \frac{c^2}{c + v}. \tag{22}$$

The speed of light in this case does not match c, but this does not mean there is a contradiction here between "principle I" and "principle II." (However, in this case "principle E" does not hold.)

If w_c in Eq. (22) is substituted for w in Eq. (21),

$$u = v + \frac{c^2}{c + v}\left(1 - \frac{v^2}{c^2}\right) = c. \tag{23}$$

This equation means that the speed of light does not depend on the velocity of the light source.

Next, if the velocity w of an object measured in S' is expressed as a times the speed of light w_c (with $0 < \mathrm{a} \leq 1$), then

$$u = v + \frac{\mathrm{a}c^2}{c + v}\left(1 - \frac{v^2}{c^2}\right) \tag{24a}$$

$$= v + \mathrm{a}(c - v), \quad 0 < \mathrm{a} \leq 1. \tag{24b}$$

$u \leq c$ holds in Eqs. (21) and (24b), and this matches with the results of experiment.

Incidentally, when deriving Eqs. (21) and (24b), a strict distinction was made between the stationary system and moving system, Therefore in these equations, $(1 - v^2/c^2)$ is not something that is first derived in the STR, and is permissible for Lorentz. $(1 - v^2/c^2)$ is a conclusion derived if an observer in the stationary system applies "principle I" to the moving system (coordinate system of the rod), and assumes "principle II" in the coordinate system of the rod. Here, there is no need to carry the assumption as far as "principle E," which acts as the foundation of the STR (see Appendix B).

VI. CONCLUSION

The established view regarding the velocity addition law is that Eq. (19) is correct. However, this paper concludes that the following equations have a reason to exist, and can coexist with Eq. (19).

$$u = v + w\left(1 - \frac{v^2}{c^2}\right), \quad 0 < w \le \frac{c^2}{c + v}. \tag{25}$$

$$u = v + \mathrm{a}(c - v), \quad 0 < \mathrm{a} \le 1. \tag{26}$$

Equations (25) and (26) derived by the author are equations employing the velocities measured by an observer in the moving system using C_A and C_{B1}.

Equation (25) is derived regarding light as a particle, and Eq. (26) is derived regarding light as a wave.

On the other hand, the equation of the STR employs the relativistic velocity $\boldsymbol{w}_r$ measured by an observer in the moving system using C_A and C_{B2}. To put it another way, the velocity addition law using the velocity $\boldsymbol{w}$ measured with C_A and C_{B1}, which are in a state of absolute synchronization, is given by Eqs. (25) and (26) derived by the author. In contrast, Eq. (19) of the STR is an equation using the relativistic velocity $\boldsymbol{w}_r$ measured with C_A and C_{B2}, which are in a relativistically synchronized state. The author has pointed out that, when velocity is measured in a moving system, the equation which can be applied by an observer in the stationary system differs depending on the sense in which the clocks used to measure the velocity match.

The historical sequence is that Eq. (19) was derived first, and Eqs. (25) and (26) in this paper were derived later. However, logically, the order should be that we derive Eq. (25) first, and then derive Eq. (19) by substituting the right side of Eq. (18) for w in Eq. (25).

Previously, when Eqs. (25) and (26) have been presented as velocity addition laws, they have been considered to be errors. However, in this paper we conclude that, although these equations are not as theoretically profound as Eq. (19), they have greater utility in practical terms than Eq. (19).

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APPENDIX A

It is possible to distinguish, experimentally, between a stationary system where light propagates isotropically, and a coordinate system where light propagates anisotropically.[5]

At the beginning, when the rod is at rest in a coordinate system where light propagates isotropically, we assume that the relation in Eq. (1) holds between the times of clocks C_A and C_B at the ends of the rod.

Next, when this rod begins to move at a constant velocity $\boldsymbol{v}$ relative to the stationary system, the times of C_A and C_{B1} must be synchronized again. At this time, if we let $\Delta t'_B$ be the time by which the time of C_{B1} at the front of the rod is delayed, then $\Delta t'_B$ takes the following value:

$$\Delta t'_{B1} = \frac{Lv}{c^2}(\mathrm{s}). \tag{A1}$$

What would happen if light propagated anisotropically in the coordinate system where the rod was at rest at the beginning?

If the rod begins to move at a constant velocity $\boldsymbol{v}$ relative to the stationary system, then Eq. (A1) will not hold. That is,

$$\Delta t'_{B1} \ne \frac{Lv}{c^2}(\mathrm{s}). \tag{A2}$$

Thus, by using the second adjustment time of the time of C_B, it is possible to predict whether or not light propagation in the coordinate system where the rod was originally at rest is isotropic. (In other words, there is a thought experience which can distinguish between a classically stationary system and a relativistically stationary system.)

Due to the above conclusion, there will naturally be some who point out that this is a breakdown of the "principle of relativity." However, the author is convinced that the cause of Eq. (A2) is that there is an unknown velocity vector accompanying the coordinate system where the rod was at rest at the beginning.

APPENDIX B

"Principle E" becomes necessary when measurement is done from the opposite standpoint.

Here, we consider the case where the observer of rod S′ moving at a constant speed measures the length of rod S placed on the x-axis in system S.(It is assumed that rod S and S′ are of the same type, and have the same length when at rest.)

Let us measure, from S, the time it takes for light emitted at a certain time from a light source at the center of the rod S′, to arrive at the rear end A and front end B of rod S′. Letting t_A be the time it takes the light to arrive at A, and t_B the time it takes the light to arrive at B,

$$t_A = \frac{1}{\gamma}\frac{L_0}{2(c + v)}, \tag{B1}$$

$$t_B = \frac{1}{\gamma}\frac{L_0}{2(c - v)}, \quad \text{however,} \quad \gamma = \frac{1}{\sqrt{1 - (v/c)^2}}. \tag{B2}$$

Next, when the travel distance ct_A for light to arrive at A is measured from S,

$$ct_A = \frac{c}{\gamma}\frac{L_0}{2(c + v)}. \tag{B3}$$

Also, the travel distance ct_{B} for light to arrive at B is

$$ct_{\mathrm{B}} = \frac{c}{\gamma}\frac{L_0}{2(c-v)}. \tag{B4}$$

Therefore, the length of the rod S′ read off from the x-axis of S at the same time in S' by observers A and B at both ends of the rod becomes the sum of (B3) and (B4), and hence

$$ct_{\mathrm{A}} + ct_{\mathrm{B}} = \gamma L_0. \tag{B5}$$

Thus, the observer in S' obtains the following value as the ratio of the length of rod S′ and rod S.

$$\text{Length of rod in } S' : \text{Length of rod in } S = \gamma L_0 : L_0. \tag{B6}$$

Next, let us still apply the "principle of relativity" to these two coordinate systems, and assume that S and S' are equivalent. In this case, an observer in S' cannot recognize the physical changes in his own coordinate system, and thus he determines, based on (B6) that rod S has contracted. That is,

$$\gamma L_0 : L_0 \rightarrow 1 : \frac{1}{\gamma}. \tag{B7}$$

The above considerations show that there are two types of Lorentz contraction, as indicated below.

Contraction I: Contraction when the length of the rod in S' treated in Section V is measured from S. Contraction in this case is true contraction, where the rod itself actually contracts.

Contraction II: Contraction (B7) when the observer in coordinate system S' regards his own system as stationary, and measures the length of a rod placed on the x-axis in S. This contraction is based on two causes: true contraction of the moving rod (contraction I) and the relativity of same time in S' introduced by Einstein.

As indicated above, the fact that the two inertial systems are equivalent in the STR means that the same measurement values will be obtained whether measurements are taken from inertial system S or S'.

Both Einstein and Lorentz had the same opinion that the length of a rod in S' contracts when observed from S. However, Lorentz did not recognize the need for further synchronization if C_{A} and C_{B} were synchronized when they were stationary.

Therefore, an observer in S' with the standpoint of Lorentz obtains the following value as the ratio of the lengths of the two rods.

$$\text{Length of rod in } S' : \text{Length of rod in } S = \sqrt{1-(v/c)^2} : 1 = 1 : \gamma. \tag{B8}$$

If the length of a rod in a stationary system is measured from S' moving at a constant velocity, then the rod in S is judged to be longer. A result like this is obtained because Lorentz compared the lengths of the two rods from the standpoint of reality.

Even if it is assumed that the two inertial systems are equivalent from the standpoint of measurement values, they cannot be regarded as the same when discussing physical quantities from the standpoint of reality.

Incidentally, S and S' are equivalent in the STR, and thus it is unacceptable for there to be different causes of rod contraction. Therefore, the STR does not go as explain the reasons why the rod contracts.

The assumption of Contraction II can be regarded as an interpretation used when physicists who support Lorentz's view of nature have attempted to somehow explain space contraction in the STR.

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