New problem of ‘twin paradox’

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Abstract

The famous paradox in special relativity is ‘twin paradox’.

It is thought that the cause of the time dilation by the watch of one of the twins who has made a space travel compared with the watch of the other who stays on the earth is ‘the asymmetry of the two coordinates caused by the accelerated motion the coordinates of the astronaut has made’.

In this paper we will consider the paradox by introducing three coordinates, and will make it clear that there exist coordinates where time dilation occurs even if asymmetry between coordinates doesn’t exist.

This paper will show there exists a condition to which the conventional explanation for ‘twin paradox’ is invalid.

I. Introduction

Since special relativity was made public, ‘twin paradox’ has been controversial. Today it is regarded as settled and is not regarded as a paradox any more.

This paper does not disagree with the prediction of special relativity, which predicts that the clock of one of the twins who has made a space travel will have slowed down as compared with the clock of the other of the twins. However, this paper has doubt about the established explanation for the reason of the time dilation.

Thus, let us reconfirm the ‘twin paradox’ for the sake of the arguments introduced in the next chapters.

Suppose the clocks initially agreeing with each other. If one clock remains at rest in an inertial frame, and the other is taken off on any sort of path and finally brought back to its starting point.

Special relativity predicts that the second clock will have slowed down as compared with the first. In today’s parlance, the astronaut will end up by becoming younger than his twin brother.

During the travel the astronaut experiences acceleration and deceleration, and so there is asymmetry between the two coordinates. Therefore, it is thought that we can’t regard that one on the earth travels and the other is at rest.

The clock of the astronaut has slowed down. Therefore, we can avoid the impossible condition where both the twins find the other younger than himself.

The explanation regarded as correct is as follows:

‘Since the coordinates of the second clock moving relative to the inertial system make accelerated motion, there exists asymmetry between the two coordinates. It is clearly the second clock that has made a motion, and so the time of the second coordinates delays.’

However,in Thought Experiment 2 in the next chapter, this paper will present the example in which this explanation is invalid.

II. 'Triplet experiment' by three observers (Thought Experiment 1) and another 'Triplet experiment' performed by replacing one observer with another observer (Thought Experiment 2)

On 'twin paradox' reconfirmed in Introduction, in order to avoid an argument concerning acceleration and deceleration that the astronaut experiences, we should deal with 'triplet experiments'.

In this chapter, therefore, we give a consideration by thought experiment dealing with triplets.

Thought Experiment 1:

Suppose the rocket A moving at the uniform velocity $3c/5$ in the plus direction along the x-axis on the earth. (c means light speed)

When the observer A in the rocket A passes before the observer M on the earth, observers M and A start their stopwatches W and W_A. (See Fig. 1a)

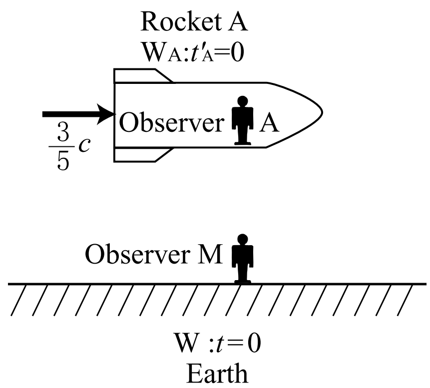

Fig. 1a The moment when the observer A in the rocket A passes before the observer M on the earth

At this point the two observers start their stopwatches W and W_A. The time passes by the watches in this paper is to be obtained applying special relativity.

All the figures in this paper, space contraction of a moving object is left out.

After that the rocket A keeps moving. When one second passes by the stopwatch W, the rocket A meets the rocket B moving from ahead. (See Fig. 1b)

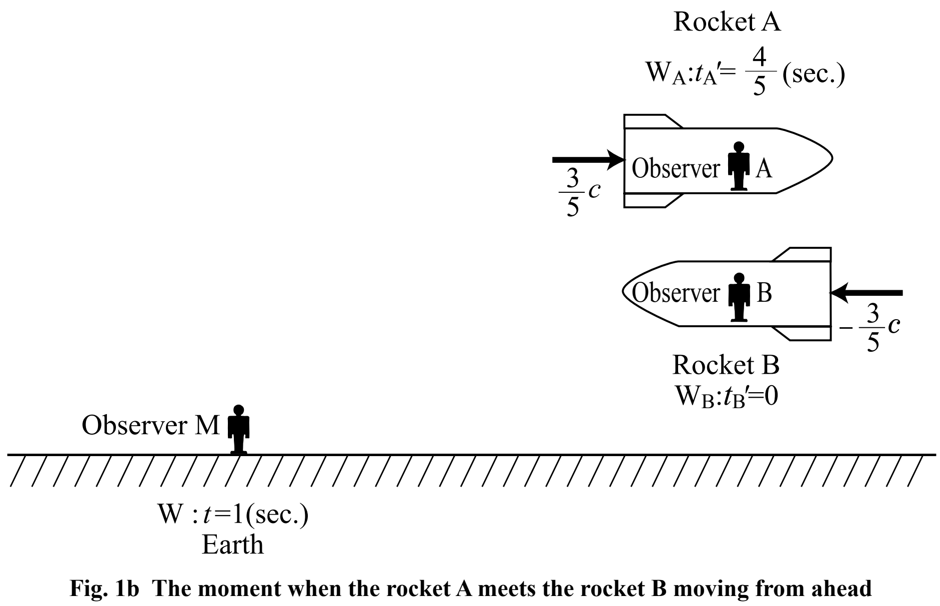

Fig. 1b The moment when the rocket A meets the rocket B moving from ahead

At this point the observer A stops his watch, while the observer B starts his watch.

At this moment, the observer A in the rocket A stops his stopwatch W_A and the observer B in the rocket B starts his stopwatch W_B. Note that the velocity of the rocket B relative to the observer M is $-3c/5$.

Suppose the time that has passed by W_A is $t_A'$and the time that has passed by W is t, we can get the following value from special relativity.

$$t_A' = (1-v^2/c^2)^{1/2}t = \gamma^{-1}t \qquad \text{(II . 1)}$$

Substituting $3c/5$ for v, and 1 for t in Eq.(II . 1), we get:

$$t_A' = 4/5 \quad \text{(sec.).} \qquad \text{(II . 2)}$$

On the other hand, the rocket B keeps moving. When the rocket B passes before the observer M, observers M and B stop their watches W and W_B. (See Fig. 1c)

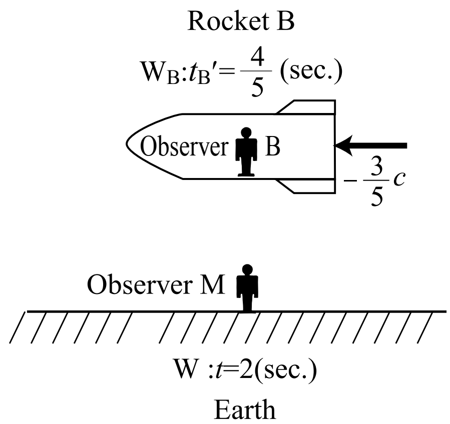

Fig. 1c The moment when the observer B in the rocket B passes before the observer M on the earth

At this point the two observers stop their watches W and W_B.

During this time, the time t which has passed by the watch W is 2(sec.).

Meanwhile, the time t_B', which has passed by watch W_B is equal to t_A', i.e., 4/5(sec.).

From these values, the relationship between t, $t_A'$and $t_B'$is:

$$t:t_A':t_B' = 2:4/5:4/5 \qquad \text{(II . 3)}$$

Expressing this relationship in another form, we get:

$$t:(t_A'+t_B') = 1:4/5 \qquad \text{(II . 4)}$$

According to special relativity, the total amount of time which has passed in the rocket A and B is smaller than the time which has passed on the earth.

From these prediction, special relativity concludes that the time in the rocket A and B passes later than the time passing on the earth.

That is, in the coordinates in 'triplets' dealt with in Thought Experiment 1, 'the rest system' is the coordinates on the earth, and 'the motion system' is the coordinates of the rockets A and B.

Thought Experiment 2:

In place of the rocket B, we introduce the rocket C moving at a different velocity from that of the rocket B.

As in Thought Experiment 1, however, when the observer A in the rocket A passes before the observer on the earth at the velocity $3c/5$, the two observers start their watches W and W_A. (See Fig. 1a)

When the time t by the watch W is 4/5(sec.), the rocket C passes before the observer

M at the velocity v'. (See Fig. 2a)

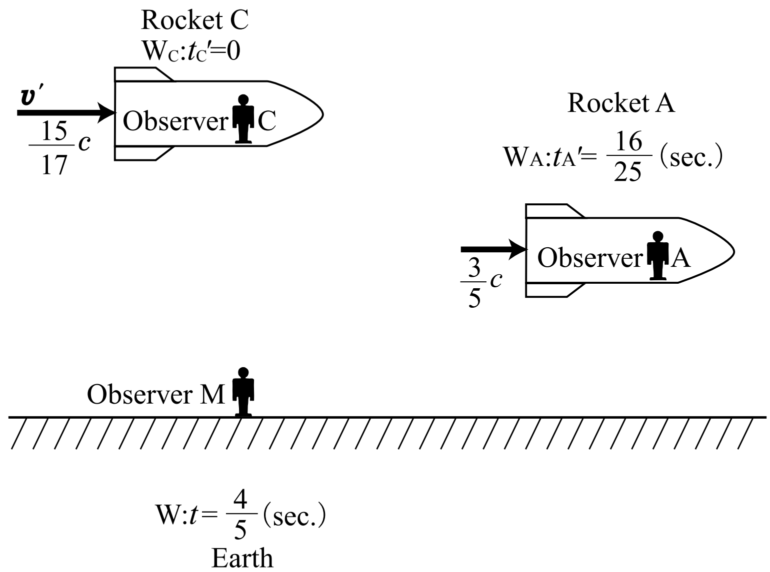

Fig. 2a When 4/5 sec. passes by the watch of the observer M on the earth, the rocket C passes before the observer M at the velocity *v′*

At this point the observer M stops his watch W, while the observer C start his watch W_C.

The velocity $v'$is the velocity of the rocket C approaching the observer A in the rocket A at the speed $3c/5$.

Next we will get the velocity of the rocket C(v') seen from the observer M.

According to the addition theorem of the velocities by special relativity, we get:

$$v' = (v_1+v_2)/(1+v_1v_2/c^2) \qquad \text{(II.5)}$$

Substituting $3c/5$ for v_1 and v_2, we get:

$$v'=15c/17 \qquad \text{(II.6)}$$

After that the rocket C keeps moving. When the rocket C catches up with the rocket A, the observer A and the observer C(in the rocket C) stop their watches W_A and W_C. (See Fig. 2b)

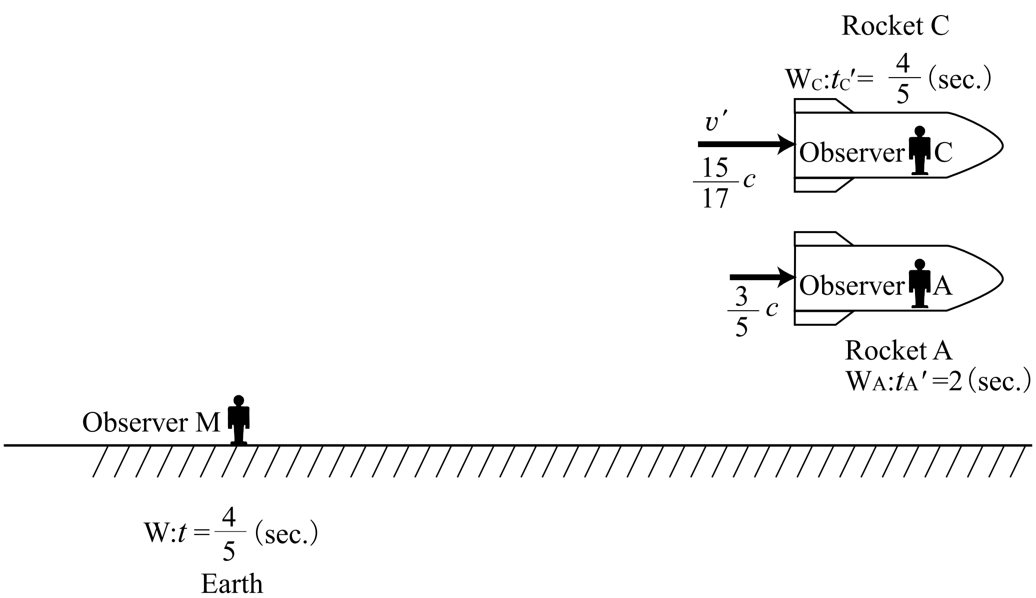

Fig. 2b The moment when the rocket C catches up the rocket A

At this point the observers A and C stop their watches.

According to special relativity, the velocity $15c/17$ of the rocket C means that the rocket C approaches the rocket A at the speed $3c/5$.

Then the observer A compares the time t_A' which has passed by his watch with the time t which has passed on the earth, and with the t_C' which has passed in the rocket C.

In order to get the values t_A' and t_C', we first get the values t_A and t_C as defined below:

Suppose the following: When the time t_A' passes by the watch W_A, the time t_A passes by the watch W. When the time t_C' passes by the watch W_C, the time t_C passes by the watch W.

At this point we can get two equations below:

$$t_A = t_C + 4/5 \quad (II.7)$$

$$vt_A = v't_C \quad (II.8)$$

Substituting t_C in Eq. (II.7) for t_C in Eq. (II.8), and $3c/5$ for v, the value in (II.6) for v', we get:

$$t_A = 5/2 \text{ (sec.)} \quad (II.9)$$

From Eq.(II. 7), we get:

$$t_C = 17/10 \text{ (sec.)} \quad (II.10)$$

Therefore, the time t_A' which passed by the watch W_A is:

$$t_A' = \gamma^{-1} t_A \quad (II.11)$$

Here, substituting 4/5(obtained from Eq.(II. 2)) for γ^{-1}, and the value of (II.9) for t_A, we get:

$$t_A' = 2 \text{ (sec.)} \quad (II.12)$$

Meanwhile, the time t_C' which has passed by the watch W_C is:

$$t_C' = \gamma'^{-1} t_C \quad (II.13)$$

Here, we get:

$$\gamma'^{-1} = (1 - v'^2/c^2)^{1/2} \quad (II.14)$$

Substituting the value of Eq.(II. 6) for v' in Eq.(II.14), we get:

$$\gamma'^{-1} = 8/17 \quad (II.15)$$

Furthermore, substituting the value of Eq.(II.10) for t_C in Eq.(II.13), we get:

$$t_C' = 4/5 \text{ (sec.)} \quad (II.16)$$

From this,

$$t_A' : t : t_C' = 2 : 4/5 : 4/5 \quad (II.17)$$

Expressing the relationship in another way, we get:

$$t_A' : (t + t_C') = 1 : 4/5 \quad (II.18)$$

As compared with the time passage in the rocket A, which is in accelerated motion and thus not properly regarded as 'the rest system', the total amount of the time which has passed on the earth and in the rocket C is smaller.

From this, special relativity concludes that the time which passes on the earth and in the rocket C delays as compared with the time which passes in the rocket A.

From the considerations above, we can say Eq.(II.3) and Eq.(II.17) are similar; the former is the equation of time dilation which the observer on the earth in Thought Experiment 1 observes as to stopwatches W_A and W_B, and the latter is the equation of time dilation which the observer in the rocket A observes as to stopwatches W(on the earth) and W_C (in the rocket C).

As a result, Thought Experiment 2 means that the observer in the rocket A has performed Thought Experiment 1(the triplet experiment) regarding himself as 'the rest system'. (See Fig. 3)

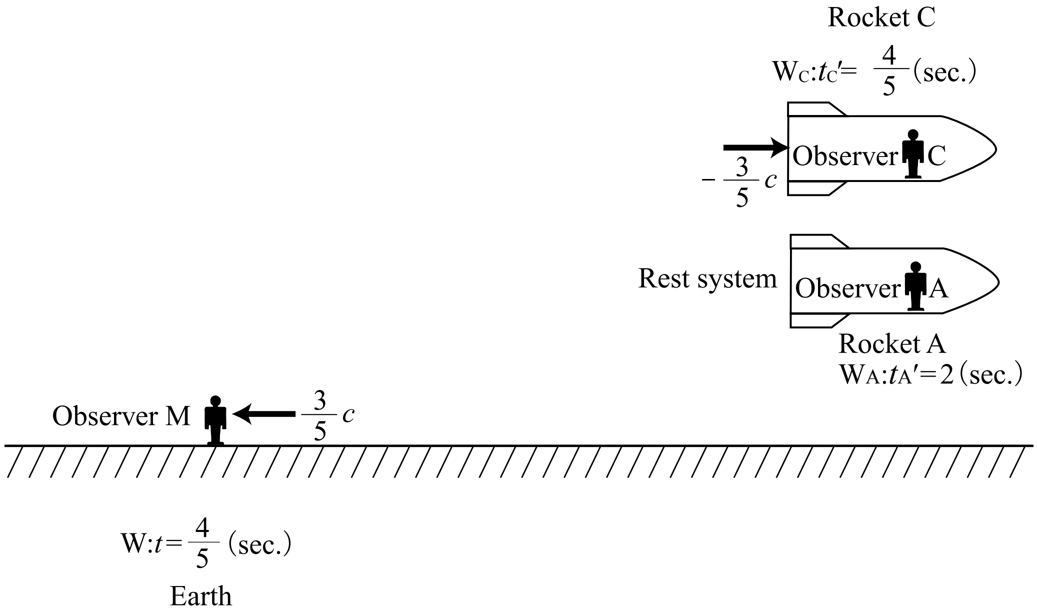

Fig. 3 The case in which Thought Experiment 2 is seen from the observer A in the rocket A

As is clear from this figure, Thought Experiment 2 corresponds to the case in which the observer A in the rocket A performs Thought Experiment 1(triplet experiment) regarding himself as 'the rest system'.

Further, we can find the coordinates of the earth in Thought Experiment 2(Fig. 3) corresponds to those of the rocket A in Thought Experiment 1. In the same way, the coordinates of the rocket C corresponds to those of the rocket B. (See Chart 1)

Thought Experiment 1 (Fig.1)		Thought Experiment 2 (Fig.3)	
Rest System	Earth	Rest System	Rocket A
Motion System	Rocket A Velocity:3c/5	Motion System	Earth Velocity:3c/5
Motion System	Rocket B Velocity:-3c/5	Motion System	Rocket C Velocity:-3c/5

Chart 1. 'Rest system' and 'motion system' in Thought Experiment 1 (Fig. 1) and Thought Experiment 2 (Fig. 3)

III.Conclusion

If the three coordinates in Thought Experiment 2 are seen from the viewpoint of whether or not accelerated motion existed in the past, 'the rest system' is the coordinates of the earth, while 'the motion system' is the coordinates of the rocket A and C.

However, when they are seen from the viewpoints of time dilation, 'the rest system' is the coordinates of the rocket A, while 'the motion system' is the coordinates of the earth and the rocket C.

In Thought Experiment 2, it is judged that the time which passes by the stopwatch on the earth delays relative to the time which passes in the coordinates of the rocket A. However, the earth didn't make the accelerated motion relative to the rocket A.

Meanwhile, the coordinates of the rocket A is judged as 'the rest system' although it performed the accelerated motion relative to the earth.

Then, what will happen if we introduce the coordinates of the rocket M(moving at uniform speed relative to the earth) and perform the same experiments as Thought Experiment 1 and 2?(Imagine by replacing the earth with the rocket M in Fig.1, 2 and 3)

We will call those thought experiments Thought Experiment 1′and 2′, which correspond to Thought Experiment 1 and 2.

In Thought Experiment 1′, where the observer in the rocket M performs, it is judged that the time which passes in the coordinates of the rocket A and B delays relative to the time which passes in the coordinates of the rocket M.

From this, the observer in the rocket M regards his coordinates as 'the rest system', while he regards the coordinates of the rocket A and B as 'the motion system'.

Meanwhile, in Thought Experiment 2′, where the observer in the rocket A performs, it is judged that the time which passes in the coordinates of the rocket M and C delays relative to the time which passes in the coordinates of the rocket A.

From this, the observer in the rocket A regards his coordinates as 'the rest system', while he regards the coordinates of the rocket M and C as 'the motion system'.

By the way, in order to perform Thought Experiment 1′and 2′by introducing the coordinates of the rocket M in place of those of the earth, the rocket M, A, B and C have to make accelerated motion beforehand, and reach the velocity which is fast enough for thought experiments to be performed.

In Thought Experiment 1′and 2′, the coordinates of the four rockets are equal in the sense that they made accelerated motion relative to the earth. There is no asymmetry among the four coordinates.

That is to say, there exists coordinates with time dilation even though all the coordinates are in accelerated motion and no asymmetry exists.

From this, we have found that there is a condition where the explanation proposed in the Introduction 'Accelerated motion made by the coordinates of the second stopwatch brings about the asymmetry, therefore the time of the coordinates of the second stopwatch which made motion delays.' can not be applied.

Therefore, we have reached the conclusion that, on 'twin paradox' a different explanation from the conventional one is needed.

Acknowledgments

I am very grateful to Takahiro Yasui for putting this paper into English.