Revealing the Essence of Planck's Constant

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Abstract

According to traditional classical quantum mechanics theory, due to the prior existence of Planck's constant, considered a universal constant, it is thought that the energy of a photon can be determined if its frequency is known, and the wavelength of a quantum can be determined if its momentum is known ($E = h\nu$ and $\lambda = h/p$).

In this paper, however, the case is made that logically, since the product of the momentum and wavelength of any photon can be expressed by the constant $p\lambda$, Planck's constant only comes into existence when $p\lambda$ is replaced with h.

In this paper, we show that Planck's constant is not a universal constant but is instead just a usual fundamental physical constant.

Abstract

Selon l'interpretation traditionelle de la théorie de la mécanique quantique classique, due à l'existence antérieure de la constante de Planck considérée comme une constante universelle, il est accepté que l'énergie d'un photon peut être déterminée si sa fréquence est connue, et la longueur d'onde d'un quantum peut être déterminée si sa quantité de mouvement est connue (avec $E = h\nu$ ou $\lambda = h/p$).

Cependant, dans cette discussion, le cas est fait que, logiquement puisque la quantité de mouvement et le produit de la quantité de movement et de la longueur d'onde de n'importe quel photon peuvent être exprimés par la constante $p\lambda$, la constante de Planck existe seulement lorsque h est exprimée comme étant égale à $p\lambda$.

Dans cette étude, nous montrons que la constante de Planck n'est pas une constante universelle, mais qu'elle est juste une constante physique ordinaire.

I. Introduction

In 1900, when deriving a formula that derived an experimental value of black-body radiation, M. Planck (1858-1947) proposed the quantum hypothesis stating that the energy of a harmonic oscillator with oscillation frequency v would quantize at the integral multiple of hv. This was the first time that Planck's constant h appeared in physics theory [1].

Planck's constant is thus thought to be a fundamental physical constant defined in the realm of quantum theory, but the essence of this constant is generally not well understood.

Below is Einstein's formula expressing the equality of energy and mass [2].

$$E = mc^2 \tag{I.1}$$

Here, m is the mass of a particle and c is the speed of light in a vacuum.

The speed of light may also be expressed through the following formula.

$$c = \lambda v \tag{I.2}$$

Meanwhile, Einstein's relational expressions regarding the energy of a photon are expressed by the following formulas:

$$E = hv \qquad \text{(I.3) [3]}$$

$$E = pc \tag{I.4}$$

Here, v is the photon's frequency and p is the photon's momentum.

The formula on which (I.4) is based is Einstein's energy-momentum relationship as below.

$$(mc^2)^2 = (m_0c^2)^2 + p^2c^2 \tag{I.5}$$

Because a photon's mass is zero, we obtain (I.4) by substituting m_0=0 in (I.5).

In other words, Formulas (I.1) and (I.4) are logically derived formulas.

Formula (I.3), on the other hand, explains an observed value and for this reason it is an experimental formula, not a logically derived formula.

If we next combine equals from Formulas (I.1) and (I.4), we obtain the following formula describing the momentum of a photon.

$$p = mc \tag{I.6}$$

If we combine equals from Formulas (I.1) and (I.3), when considering Formula (I.6), we obtain Einstein's following formula [4].

$$p = \frac{h}{\lambda} \tag{I.7}$$

This formula expresses the relationship between a photon's momentum and wavelength. L. de Broglie (1892-1987) used the above logic of the derived Formula (I.7) in predicting that the relation between an electron's momentum and wavelength can be expressed, as in (I.7) [5].

Meanwhile, A. Einstein (1879-1955) showed that the momentum of light quanta can be represented by the following formula [6].

$$p = \frac{h\nu}{c} \tag{I.8}$$

Einstein showed that when considering Formula (I.2), Formula (I.7) can be derived from Formula (I.8) and Formula (I.4) can be derived from Formulas (I.3) and (I.8).

Incidentally, we obtain the following when expressed in mathematically inverse proportions.

$$xy = \mathrm{a} \tag{I.9}$$

Comparing Formulas (I.7) and (I.9), x and y correspond to p and λ and a correspond to h.

This supports the concept that Planck's constant is no more than a simple constant.

Thus, using non-historic reasoning, the true essence of this constant is revealed in this paper.

Beforehand, let us verify the following points regarding fundamental physical constants and Plank's constant.

Fundamental physical constants play an essential part in elementary formulas that describe natural phenomena and can be largely divided into universal constants and material constants.

Also, physical quantities and constants are included in fundamental physical constants that belong to one category.

Physical quantities belonging to micro material constants include electron mass m_e, elementary charge e, and electron Compton wavelength λ_C, and include such constants as the fine-structure constant α and the Rydberg constant R_∞.

The Boltzmann constant k and the Avogadro constant N_A are examples of macro material constants.

However, Planck's constant h is thought to be a fundamental constant representative of quantum mechanics and considered to be a universal constant.

Because Plank's constant has an action quantity dimension, it was at first called an action quantum when quantum theory originally emerged. h appears in the inequality $\Delta x \Delta p_x \geqq \hbar/2$ when W. Heisenberg (1901-1976) discovered the uncertainty principle in 1927.

Planck's constant h, along with the speed of light in a vacuum c and the Newtonian constant of gravitation G, also plays an important role when assembling planck units from universal constants.

From the above, Planck's constant is a constant by name, but it has come to be strongly regarded as being the smallest unit of angular momentum.

Planck's constant is currently measured by the watt balance method.

Among others, this research is being performed by NPL and NIST [7], [8].

The estimated value of Planck's constant according to 1998 CODATA was determined based on data measured using the watt balance method.

A method that determines h from only directly measured quanta, this experiment was thought to be able to obtain the best value for h.

Another way to measure Planck's constant is the X-ray crystal density (XRCD) method.

This method measures the Avogadro constant N_A using a virtually pure single crystal such as silicon. NMIJ and PTB have recently reported extremely accurate measurement results [9], [10].

Because Planck's constant and the Avogadro constant are closely related to each other, a more accurate measurement of h is possible if a more accurate measurement of N_A is obtained.

The CODATA Recommended Values of the Fundamental Physical Constants (2002) used these measurement results and the measurement results of the abovementioned watt balance method as a basis to determine Planck's constant [11].

Similar progress is made every year in measuring Planck's constant, but little progress has been made in understanding the essence of this constant. Therefore, in this paper we attempt to reveal the essence of this constant.

II. Deriving Planck's Constant from Fundamental Physical Constant

If m_e is the mass of an electron, an electron's mass energy E_0 can be represented by the following formula.

$$E_0 = m_e c^2 \tag{II.1}$$

Meanwhile, if ν_C is the frequency of a photon carrying an amount of energy equivalent to E_0, the following is true (See Appendix).

$$E_0 = h\nu_C \tag{II.2}$$

Combining equals from Formulas (II.1) and (II.2), we obtain:

$$m_e c^2 = h\nu_C \tag{II.3}$$

Fundamentally these two types of energy have different characteristics, but from a quantitative perspective, it is possible to combine them as equals.

Thus, a photon's frequency ν_C is expressed as follows.

$$\nu_{\mathrm{C}} = \frac{m_{\mathrm{e}}c^2}{h} \tag{II.4}$$

Next, a photon's wavelength λ becomes:

$$\begin{aligned} \lambda &= \frac{c}{\nu_{\mathrm{C}}} \\ &= \frac{h}{m_{\mathrm{e}}c} \end{aligned} \tag{II.5}$$

Now, an electron's Compton wavelength λ_{C} is represented by the following formula.

$$\lambda_{\mathrm{C}} = \frac{h}{m_{\mathrm{e}}c} \tag{II.6}$$

The wavelength of a photon with energy E_0 is the same as the Compton's wavelength λ_{C} of an electron.

Thus, (II.1) can be transformed as follows.

$$\begin{aligned} E_0 &= m_{\mathrm{e}}c^2 \\ &= m_{\mathrm{e}}c\lambda_{\mathrm{C}}\nu_{\mathrm{C}} \end{aligned} \tag{II.7}$$

In (II.7), λ_{C} is the wavelength of a photon, not an electron. However, because the right sides of (II.7) and (II.3) match, the following relationship holds true in the case of a photon as well.

$$m_{\mathrm{e}}c\lambda_{\mathrm{C}} = h \tag{II.8}$$

Next we apply the same logic on protons that was applied on electrons.

Repeating the same logic on protons as was applied on electrons, we finally derive the following formula.

$$m_{\mathrm{p}} c \lambda_{\mathrm{C,p}} = h \tag{II.9}$$

Here, m_{p} is the proton's mass, $\lambda_{\mathrm{C,p}}$ is the proton's Compton's wavelength as well as the photon's wavelength, which has the same value

III. Planck's Constant Derived from the Various Energies of a Photon

The specific energy held by a photon was considered in the previous chapter. This chapter is a more generalized discussion based on a photon having various types of energy.

First, by generalizing (II.3) we obtain the following:

$$mc^2 = h\nu \tag{III.1}$$

Here, m is not necessarily the entire mass of the electron. The mass of the electron is being gradually reduced due to the emission of photons, and m corresponds to the reduced mass of that electron. (when $0 < m$)

In other words, (III.1) is saying that the reduction in electron energy is equal to the energy of the emitted photons.

The current reduced mass m is defined as follows.

$$m = \mathrm{a} m_{\mathrm{e}} \quad (\text{when } 0 < \mathrm{a}) \tag{III.2}$$

The momentum of a photon emitted from the electron at this time is expressed as follows.

$$\begin{aligned} p &= mc \\ &= \mathrm{a} m_{\mathrm{e}} c \end{aligned} \tag{III.3}$$

Also, since there is an inverse proportional relationship between a photon's momentum and wavelength, the wavelength of this photon is can be expressed by the following formula.

$$\lambda = \frac{\lambda_C}{a} \tag{III.4}$$

Thus, the product $p\lambda$ of an emitted photon's momentum and wavelength is:

$$\begin{aligned} p\lambda &= mc\lambda \\ &= (am_e c)(\frac{\lambda_C}{a}) \\ &= m_e c\lambda_C \end{aligned} \tag{III.5}$$

We can see that ultimately, the product $p\lambda$ of the momentum and wavelength of any photon is the same as the constant $m_e c\lambda_C$. That is,

$$\begin{aligned} p\lambda &= m_e c\lambda_C \\ &= h \end{aligned} \tag{III.6}$$

Considering (III.6), it is possible to logically derive (I.3) from (I.1).

Thus,

$$\begin{aligned} E &= mc^2 \\ &= mc\lambda\nu \\ &= h\nu \end{aligned} \tag{III.7}$$

IV. Discussion

We next substitute the following values for physical quantities in $m_e c\lambda_C$ [11].

$$m_e = 9.1093826 \times 10^{-31} \text{ kg} \tag{IV.1}$$

$$c = 2.99792458 \times 10^{8} \text{ m} \cdot \text{s}^{-1} \tag{IV.2}$$

$$\lambda_C = 2.426310238 \times 10^{-12} \text{ m} \tag{IV.3}$$

By doing so, $m_e c\lambda_C$ becomes as follows.

$$m_e c\lambda_C = 6.6260693 \times 10^{-34}\ \ \mathrm{J \cdot s} \qquad \text{(IV.4)}$$

Meanwhile, Planck's constant has the following value [11].

$$h = 6.6260693 \times 10^{-34}\ \ \mathrm{J \cdot s} \qquad \text{(IV.5)}$$

$m_e c\lambda_C$ and h are a perfect match.

The currently known values for m_e or λ_C were not determined through experimentation.

m_e was determined through precise calculations from Rydberg constant formulas, and λ_C was obtained by substituting m_e in the formula $\lambda_C = h/m_e c$.

Based on measured data from theoretical formulas or experiments designed to represent the fundamental laws of physics, many fundamental physical constants are being adjusted to avoid conflicts from arising between these constants.

Because the formula to determine an electron's Compton wavelength is $\lambda_C = h/m_e c$, naturally the modified version of this Formula (II.8) is true.

However, λ_C in (II.8) is not the wavelength of an electron, but is the wavelength of a photon with the same value as the electron's Compton wavelength. Also, it is important to note that $m_e c$ is not the momentum of an electron but the momentum of a photon.

Logically, however, Planck's constant should thought of as a constant that only comes into existence once Formula (I.1) is rewritten into Formula (I.3) to include a photon's frequency, and the subsequent recognition that the non-frequency components $mc\lambda$ form a constant which can be replaced by h.

In other words, (II.8) can be interpreted to mean not "$m_e c\lambda_C$ and h are identical" but instead to mean "$m_e c\lambda_C$ is h."

Therefore, this paper does not claim the discovery of any relationship in (II.8).

It simply defines that because the values of $m_e c\lambda_C$ and $m_p c\lambda_{C,p}$ are the same in (II.8) and (II.9), this value should hereafter be called Planck's constant h.

Rather than naming this constant as Planck's constant h, we can simply regard it as $m_e c\lambda_C = m_p c\lambda_{C,p} = p\lambda =$const.

However, because this constant has been historically used in others of Planck’s research, it has taken on the image of being a discovered universal constant.

V. Conclusion

1. Planck’s constant *h* only came into existence once it was defined

According to existing theory, Formulas (I.1) and (I.3) have been thought to have similar importance. However, according to our discussion, (I.1) is the more fundamental of the two.

Formula (I.3) is Formula (I.1) rewritten to also include frequency. The right side of (III.7), the product of the physical quantities $mc\lambda$ except for frequency, is a steady value.

Regardless of whether $m_e c\lambda_C$ or $m_p c\lambda_{C,p}$ are called Planck’s constant h, in this paper we conclude that Planck’s constant h only came into existence once it was defined.

However, not being aware of what should have been defined, this task was skipped, and thus Planck’s constant was believed to be a discovered universal constant.

Thus, it is valid to regard Planck’s constant not as a universal constant but as a physical constant on par with the fine structure constant α or the Rydberg constant R_∞.

2. Formula (I.1) is the more fundamental than Formula (I.3)

As mentioned in the Introduction, the thought process behind Einstein’s derivation of Formula (I.7) is as follows.

$$\begin{array}{ccccc} \text{(I.3)}\quad E = h\nu & — & p = \dfrac{h\nu}{c} \quad \text{(I.8)} & & \\ \Downarrow & & \Downarrow \leftarrow c = \lambda\nu \quad \text{(I.2)} & & \text{(V.1)} \\ \text{(I.4)}\quad E = pc & , & p = \dfrac{h}{\lambda} \quad \text{(I.7)} & & \end{array}$$

Meanwhile, de Broglie combined equals from Formulas (I.1) and (I.3) and further derived Formula (I.7) when considering Formula (I.6).

Namely,

$$
\begin{array}{l}
\text{(I.1)} \quad E = mc^2 \quad — \quad E = h\nu \quad \text{(I.3)} \\
\text{(I.6)} \quad p = mc \rightarrow \Downarrow \\
\qquad pc = p\lambda\nu = h\nu \leftrightarrow p = \dfrac{h}{\lambda} \quad \text{(I.7)}
\end{array}
\qquad \text{(V.2)}
$$

However, Formula (I.3) was derived according to the following process in this paper.

$$
\begin{array}{l}
(mc^2)^2 = (m_0c^2)^2 + p^2c^2 \quad \text{(I.5)} \\
\quad \updownarrow \qquad\qquad\qquad \Downarrow \leftarrow m_0 = 0 \\
\text{(I.1)} \quad E = mc^2 \quad — \quad E = pc \quad \text{(I.4)} \\
\qquad\qquad\qquad \Downarrow \\
\qquad\qquad p = mc \quad \text{(I.6)} \\
\qquad\qquad\qquad \Downarrow \\
p\lambda = mc\lambda = m_{\mathrm{e}}c\lambda_{\mathrm{C}} = \underline{\mathrm{const}} = h \leftrightarrow p = \dfrac{h}{\lambda} \quad \text{(I.7)} \\
\qquad\qquad\qquad \Downarrow \\
E = mc^2 = mc\lambda\nu = h\nu \quad \text{(I.3)}
\end{array}
\qquad \text{(V.3)}
$$

When comparing Formula (I.3), Formulas (I.7) and (I.8), historically, Formulas (I.3) and (I.8) were obtained before Formula (I.7). Thus, Einstein and de Broglie presumed Formulas (I.3) and (I.8) when deriving Formula (I.7).

However, the supposition of this paper is that because logically $p\lambda = h$ is true, Formula (I.3) can be derived.

Formulas (I.1) and (I.3) are traditionally thought to be representative formulas of the theories of special relativity and quantum mechanics, the roots of modern physics, and these two formulas have been thought to have similar importance. However, according to our discussion, Formula (I.1) is the more fundamental of the two.

Appendix

The reviewer of this paper made the following comment.

The "proof" of the paper is based on the assumption that an electron at rest can decay into a single photon, which is in violation of conservation of momentum. Further, conservation of electric charge prohibits decay of an electron into any number of photons. Thus, the decay the author seeks to investigate cannot occur.

The author is sufficiently mindful of the point being made in the reviewer's comments regarding the current revision of this paper, but a slight supplement is added to avoid any doubt or confusion.

The author is not asserting in this paper that the rest mass energy of an electron decays into a single photon.

While the energy of naturally existing photons carry a variety of values, this paper happens to use an example of what would happen to the wavelength of a photon if it had the same energy as E_0 .

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