Deriving Planck's Constant from Fundamental Physical Constant

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Abstract

Planck's constant h is equivalent to $m_e c\lambda_c$, the product of an electron's rest mass m_e, speed of light c, and an electron's Compton wavelength λ_c.

Furthermore, we can see that $p\lambda$, the product of the momentum of all quanta and the wavelength associated with that quantum, always takes a constant value. Without fully understanding the essence of this fixed value, it has been called Planck's constant.

Discussion

In 1900, when deriving a formula that derived an experimental value of black-body radiation, Planck proposed the quantum hypothesis stating that the energy of a harmonic oscillator with oscillation frequency υ would quantize at the integral multiple of $h\upsilon$. This was the first time that Planck's constant appeared in physics theory.

Planck's constant is thus thought to be a fundamental physical constant defined in the realm of quantum theory, but the nature of this constant is generally not well understood.

In this discussion we consider whether Planck's constant, a fundamental physical constant, can be explained using other basic physical quantities, and we reveal the essence of this constant.

First, Einstein's special relativity equation showing energy-mass equivalence is expressed as follows.

$$E= mc^2 \tag{1}$$

Einstein's relational expressions regarding light quanta are expressed by the following formulas.

$$E= h\upsilon \tag{2}$$

$$E=pc \tag{3}$$

Formula (1) represents the theory of special relativity, while Formulas (2) and (3) are important fundamental formulas of quantum mechanics.

We consider the case in which all the rest mass energy of a single electron currently at rest in free space is emitted as one photon.

In this case, the following formula shows the relationship between the photon's energy and its wavelength λ, and oscillation frequency υ, when expressed as m_e the rest mass of the electron.

$$\begin{aligned} m_e c^2 &= m_e c \lambda \upsilon \\ &= pc \\ &= p \lambda \upsilon \end{aligned} \qquad (4)$$

Here, $m_e c$ is the momentum of the emitted photon.

The following relationship is also true.

$$\begin{aligned} m_e c^2 &= h\upsilon \\ &= hc/\lambda \end{aligned} \qquad (5)$$

From the first formula of the right side of Formula (4) and the first formula of the right side of Formula (5), we can express the wavelength of the emitted photon according to the following formula.

$$\lambda = h/m_e c \qquad (6)$$

Here, we can see that by definition, the wavelength of the emitted photon is equivalent to the electron's Compton wavelength λ_c.

Also, from the second formula of the right side of Formula (4) and the second formula of the right side of Formula (5), we can derive the following de Broglie equation.

$$p = h/\lambda \qquad (7)$$

The de Broglie equation is normally used to find the momentum of a particle from that particle's wavelength, or to find the associated wavelength of a particle from that particle's momentum.

However, an electron's mass, speed of light, and an electron's Compton wavelength are all physical quantities that can be directly measured through experimentation, but Planck's constant is a constant that is only indirectly determined from experiments.

Thus, in this discussion, we predict that the Compton wavelength is a fundamental constant that is more essential than Planck's constant.

Because Formula (2) is historically derived before Formula (7), when we use Formula (7) to find the momentum or wavelength of a particle, we are nonchalantly using Planck's constant *h*.

Theoretically, however, we should be determining Planck's constant from the observed values m_e, c, and λ_c.

In other words, instead of using Planck's constant to derive the Compton wavelength, we should be using the Compton wavelength to derive Planck's constant.

Here, we rewrite Formula (6) according to this discussion as below.

$$m_e c \lambda_c = \text{const}$$
$$= h \qquad (8)$$

Next, we substitute the following values for m_e, c, and λ_c.

$$m_e = 9.1093826 \times 10^{-31} \ \text{kg} \qquad (9)$$
$$c = 2.99792458 \times 10^{8} \ \text{m/s} \qquad (10)$$
$$\lambda_c = 2.426310238 \times 10^{-12} \ \text{m} \qquad (11)$$

The resulting constant in Formula (8) is as follows.

$$m_e c \lambda_c = 6.626069346 \times 10^{-34} \ \text{J} \cdot \text{s} \qquad (12)$$

The currently adopted Planck's constant has the following value.

$$h = 6.6260693 \times 10^{-34} \ \text{J} \cdot \text{s} \qquad (13)$$

The two are of course the same.

Because Formula (1) is true for a photon with all possible oscillation, however, we can generalize Formula (7) to also be true for all quanta having various momentums and wavelengths. That formula is as shown below.

$$p\lambda = \text{const}$$
$$= h \qquad (14)$$

Formula (14) can be understood as the $p\lambda$, the product of the momentum of quanta such as photons, electrons, or protons and wavelength of the waves associated with such quanta, to always be the constant h.

Also because $p\lambda$ is always constant, if we substitute this value for h in Formula (4), we obtain Formula (2) which shows that the photon's energy is proportional to oscillation υ.

The important fact here is that $p\lambda$ is always constant.

Until now, we have been deriving Formula (7) using Formula (1) and Formula (2), when what we should have been doing is deriving Formula (2) and Formula (7) from Formula (1).

Formula (1) and Formula (2) are traditionally thought to be representative formulas of the theories of special relativity and quantum mechanics, the roots of modern physics, and these two formulas have been thought to have similar importance. However, according to our discussion, Formula (1) is the more fundamental of the two.

Based on the clue that Planck's constant is equivalent to $m_e c \lambda_c$, the product of three fundamental physical constants, in this discussion we derived the generalized relationship of Formula (14).

We also found that the product of the momentum of all quanta and the wavelength associated with that quantum, $p\lambda$, is always a constant value. We should reconsider whether this fixed value should be treated as a fundamental physical constant.

However, we currently regard this constant as a fundamental physical constant and call it Planck's constant h.

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