

The possibility of the existence of an unknown hydrogen atom energy level

Koshun Suto^{a)}

5-24, Oote-Town, Isesaki City 372-0048, Japan

(Received 3 April 2009; accepted 24 June 2009; published online 27 August 2009)

Abstract: It is theoretically possible for a hydrogen atom to have an energy level that is even lower than the ground state. Assuming that hydrogen atoms with this energy do exist, it would be also necessary to adjust the Rydberg formula, which predicts the wavelength of photons emitted during an electron transition. When solving for the photon wavelength using the formula newly derived in this paper, there is a slight discrepancy between the theoretical value and the experimental value thought to completely match this wavelength. This paper asserts the possibility of the existence of a hydrogen atom energy level even lower than the ground state, and concludes that an investigation of this is necessary. © 2009 *Physics Essays Publication*. [DOI: 10.4006/1.3177453]

Résumé: D'un point de vue théorique, l'atome d'hydrogène peut prendre des états énergétiques plus bas que son état fondamental. Si nous supposons que des atomes d'hydrogène à ces états énergétiques existent, alors des rectifications au niveau de la formule de Rydberg, servant à calculer les longueurs d'ondes des photons émis lors des transitions électroniques, deviennent nécessaires. Nous avons introduit dans ce rapport, une nouvelle formule permettant de calculer les longueurs d'ondes des photons; ces valeurs se situaient alors dans des intervalles extrêmement fines entre les valeurs théoriques et celles expérimentales. Ainsi, il est d'un grand intérêt de vérifier l'existence d'atome d'hydrogène se trouvant à des états énergétiques plus bas que son état fondamental.

Key words: Hydrogen Atom; Classical Quantum Theory; Classical Electron Radius; Rydberg Formula; Balmer Series; Zero-Point Energy.

I. INTRODUCTION

The author's previous paper asserted that the potential energy of a hydrogen atom $V(r)$ is equal to the reduction in an electron's rest state mass, or $-\Delta m_e c^2$, and that this value has a lower limit¹

$$V(r) = -\Delta m_e c^2, \quad (1)$$

$$0 \geq V(r) \geq -m_e c^2. \quad (2)$$

Because these two formulas are important formulas in this paper as well, we revisit the theory used to derive these formulas.

Let us imagine a single electron that is at rest in a macroscopic space and thus holds only rest mass energy $m_e c^2$. Let us assume that this electron at rest is attracted to the proton; in other words, it is attracted to the atomic nucleus of the hydrogen atom.

During this time, when the electron transit to a lower energy level and kinetic energy (K) increases, an amount of energy equaling the increased kinetic energy is released outside the atom. In order to maintain the law of energy conservation, an energy source is needed to supply the increased kinetic energy and released photonic energy $\hbar\omega$. In this case, the energy that was originally held by the electron is only the rest mass energy.

Therefore, the law of energy conservation in this case can be written as

$$-\Delta m_e c^2 + K + \hbar\omega = 0, \quad K = \hbar\omega. \quad (3)$$

From this and Eq. (A5b), we are able to determine the following lower limit of the energy of a hydrogen atom (see Appendix A):

$$E = \frac{V(r)}{2} \quad (4a)$$

$$= -\frac{m_e c^2}{2} \quad (4b)$$

$$= -0.255 \text{ MeV}. \quad (4c)$$

Meanwhile, according to the classical quantum theory, the total energy of a hydrogen atom is

$$E_n = -\frac{1}{2} \left(\frac{1}{4\pi\epsilon_0} \right)^2 \frac{m_e e^4}{\hbar^2 n^2}, \quad n = 1, 2, \dots \quad (5)$$

Here, n is a principal quantum number.

Equation (5) represents the ground state energy of a hydrogen atom when n is 1, and this energy is $E_1 = -13.6 \text{ eV}$.

Here, if we redefine the energy of a hydrogen atom when its energy is $-m_e c^2/2$ as its "new ground state energy" and represent this as E_{ng} , then these two energy values have the following relationship:

^{a)}koshun_suto129@mbr.nifty.com

$$\frac{E_1}{E_{ng}} = -\frac{1}{2} \left(\frac{1}{4\pi\epsilon_0} \right)^2 \frac{m_e e^4}{\hbar^2} \times \left(-\frac{2}{m_e c^2} \right) \quad (6a)$$

$$= \left(\frac{e^2}{4\pi\epsilon_0 \hbar c} \right)^2 \quad (6b)$$

$$= \alpha^2 \quad (6c)$$

$$= 5.325 \times 10^{-5}, \quad (6d)$$

where α is a fine structure constant.

This results in a considerable discrepancy in the available energy according to theory, or E_{ng} , and the actual ground state energy, or E_1 .

According to quantum mechanics, the first quantum number is $n=1$, but for this paper we assume the presence of another state where $n=0$. Then, we use H_0 to represent a hydrogen atom in this state.

I believe that there are three positions that can be taken here.

- (1) The theoretical lower limit for the energy of a hydrogen atom is $-m_e c^2/2$. However, the minimum energy value as determined through experimentation is E_1 or -13.6 eV. Therefore, there is no energy state H_0 for a hydrogen atom that is even lower than the ground state.
- (2) Because the lower limit value for the energy of a hydrogen atom is $-m_e c^2/2$, H_0 may exist for hydrogen atoms. And when considering that earnest discussion on this problem has not been made in the past, it is not an appropriate scientific attitude to simply assert that H_0 does not exist.
- (3) Because the lower limit value for the energy of a hydrogen atom is $-m_e c^2/2$, H_0 must exist.

I would like to make it clear that this paper is taking position (2), not position (3).

II. FORMULAS CONTAINING ENERGY EVEN LOWER THAN E_1

Even when stating the existence of a lower limit for the energy of a hydrogen atom, this does not mean that the existence of this energy level can always be confirmed. We should determine whether or not this energy level exists by checking whether the photonic energy expected according to theory is actually consistent with that obtained through experimentation.

If we are able to confirm the existence of a hydrogen atom with $-m_e c^2/2$ energy, it would be relatively simple to derive a formula that includes this energy level.

Incidentally, even if H_0 exists, $E=-\infty$ when $n=0$ as long as we use Eq. (5), thereby making it impossible to determine the energy of H_0 .

Thus, for this paper, it is necessary to derive a separate equation that is equivalent to Eq. (5) when $n \geq 1$ and that gives a value that is not infinite when $n=0$.

Incidentally, to derive Eq. (5), r_n from Eq. (A7a) is substituted into Eq. (A8).

Also, Bohr used the following quantum condition when deriving Eq. (A7a) for the orbit radius of a hydrogen atom:

$$p_n \times 2\pi r_n = 2\pi n \hbar, \quad n = 1, 2, \dots \quad (7)$$

Like Bohr's quantum condition, the quantum condition used in this paper cannot be theoretically determined. Thus, we estimate a new quantum condition to substitute into Eq. (7) for this paper and use this to derive an energy equation that also includes E_{ng} .

Next, when deriving the new equation for this paper, it is necessary to satisfy one of the following conditions for when n is 0 or 1:

Condition 1:

$$\text{When } n=0: r_0 = r_c, \quad \text{when } n=1: r_1 \doteq a_B. \quad (8)$$

Condition 2:

$$\text{When } n=0: r_0 = r_c, \quad \text{when } n=1: r_1 = a_B. \quad (9)$$

Here, r_c is the classical electron radius (see Appendix B).

When deriving an equation that satisfies these conditions, the following two conditions can be predicted as simple quantum conditions:

$$p_n \times 2\pi r_n = 2\pi(\alpha + n)\hbar, \quad n = 0, 1, 2, \dots, \quad (10)$$

$$p_n \times 2\pi r_n = 2\pi[\alpha + (1 - \alpha)n]\hbar, \quad n = 0, 1, 2, \dots \quad (11)$$

Multiplying both sides of Eq. (A2) by r^2 and using the quantum conditions of Eqs. (10) and (11), we obtain the following two equations:

$$r_n = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} (\alpha + n)^2, \quad n = 0, 1, 2, \dots, \quad (12)$$

$$r_n = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} [\alpha + (1 - \alpha)n]^2, \quad n = 0, 1, 2, \dots \quad (13)$$

Of these equations, Eq. (12) satisfies condition 1 and Eq. (13) satisfies condition 2.

Furthermore, when we substitute the radii from Eqs. (12) and (13) into Eq. (A8), we obtain the following two equations:

$$E_n = -\frac{1}{2} \left(\frac{1}{4\pi\epsilon_0} \right)^2 \frac{m_e e^4}{\hbar^2} \frac{1}{(\alpha + n)^2}, \quad n = 0, 1, 2, \dots, \quad (14)$$

$$E_n = -\frac{1}{2} \left(\frac{1}{4\pi\epsilon_0} \right)^2 \frac{m_e e^4}{\hbar^2} \frac{1}{[\alpha + (1 - \alpha)n]^2}, \quad n = 0, 1, 2, \dots \quad (15)$$

If we then expand the denominators of Eqs. (14) and (15), we obtain the following for each of these:

$$(\alpha + n)^2 = \alpha^2 + 2\alpha n + n^2, \quad (16)$$

$$[\alpha + (1 - \alpha)n]^2 = \alpha^2 + 2\alpha(1 - \alpha)n + (1 - \alpha)^2 n^2. \quad (17)$$

$2\alpha n$ in Eq. (16) is too large to be ignored by n^2 , and $2\alpha(1 - \alpha)n$ in Eq. (17) is too large to be ignored by (1

$-\alpha)^2 n^2$. For this reason, the theoretically derived photonic energy or wavelength greatly differs from the actual obtained value.

We therefore abandon the quantum conditions of Eqs. (10) and (11) in this paper and adopt the following quantum condition:

$$p_n \times 2\pi r_n = 2\pi\hbar\sqrt{\alpha^2 + n^2}, \quad n = 0, 1, 2, \dots \quad (18)$$

Since $\alpha^2 = 5.325 \times 10^{-5}$,

$$\sqrt{\alpha^2 + n^2} \doteq n. \quad (19)$$

Multiplying both sides of Eq. (A2) by r^2 and this time using the quantum condition of Eq. (18), we obtain the following:

$$r_n = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} (\alpha^2 + n^2) \quad (20a)$$

$$= a_B (\alpha^2 + n^2) \quad (20b)$$

$$= r_c + a_B n^2 \quad (20c)$$

$$= r_c \left(1 + \frac{n^2}{\alpha^2}\right), \quad n = 0, 1, 2, \dots \quad (20d)$$

By substituting this r_n into Eq. (A8), we obtain the following:

$$E_n = -\frac{1}{2} \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{m_e e^4}{\hbar^2} \frac{1}{\alpha^2 + n^2} \quad (21a)$$

$$= -2\pi\hbar c R \frac{1}{\alpha^2 + n^2} \quad (21b)$$

$$= -\frac{2\pi\hbar c R}{\alpha^2} \frac{\alpha^2}{\alpha^2 + n^2} \quad (21c)$$

$$= -\frac{m_e c^2}{2} \frac{1}{1 + n^2/\alpha^2}, \quad n = 0, 1, 2, \dots \quad (21d)$$

Here, R is the Rydberg constant, which is defined by the following equation:

$$R = \frac{1}{4\pi} \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{m_e e^4}{c\hbar^3}. \quad (22)$$

In Eq. (21d), H_0 has limited energy, and this resembles the harmonic oscillator zero-point energy.

Incidentally, according to Bohr's classical quantum theory, the photonic energy emitted during a transition between energy levels ($E_n - E_m$) and wavelength λ for principal quantum numbers m and n can be expressed as follows:

$$E_n - E_m = 2\pi\hbar c R \left(\frac{1}{m^2} - \frac{1}{n^2}\right), \quad m \geq 1, \quad n = m + 1, m + 2, \dots, \quad (23)$$

$$\frac{1}{\lambda} = R \left(\frac{1}{m^2} - \frac{1}{n^2}\right), \quad m \geq 1, \quad n = m + 1, m + 2, \dots \quad (24)$$

TABLE I. Balmer series' spectral theoretical and experimental values from Sommerfeld's textbook.

n	Theoretical value (nm)	Experimental value (nm)
3	656.280	656.280
4	486.138	486.133
5	434.051	434.047
6	410.178	410.174
7	397.011	397.006
8	388.909	388.900
9	383.543	383.538

In this paper, however, we use Eq. (18) instead of Bohr's quantum condition.

By doing so, the energy emitted when the principal quantum number transitions from n to m is expressed as follows:

$$E_n - E_m = 2\pi\hbar c R \left(\frac{1}{\alpha^2 + m^2} - \frac{1}{\alpha^2 + n^2}\right) \quad (25a)$$

$$= \frac{m_e c^2}{2} \left(\frac{1}{1 + m^2/\alpha^2} - \frac{1}{1 + n^2/\alpha^2}\right), \quad m \geq 0, \quad n = m + 1, m + 2, \dots \quad (25b)$$

The wavelength λ of the photon corresponding to this energy is expressed as follows:

$$\frac{1}{\lambda} = R \left(\frac{1}{\alpha^2 + m^2} - \frac{1}{\alpha^2 + n^2}\right), \quad m \geq 0, \quad n = m + 1, m + 2, \dots \quad (26)$$

III. DISCUSSION

(1) For the Balmer series of a hydrogen atom, $m=2$. The German language textbook published by Sommerfeld in 1919 presumes that there is a perfect match between the Balmer series' spectral theoretical and experimental values (see Table I).

Based on this table, this paper does not disagree with the conventional view that the theoretical and experimental values should be considered the same.

However, modern technological advances enable us to measure the physical constants with more accuracy than was possible at that time.

The experimental and theoretical values for the photon wavelength as obtained from Eqs. (24) and (26) are summarized in the following table (see Table II).

For wavelength value calculations, the following values were used for R and α from CODATA:²

$$R = 10\,973\,731.568\,527 \text{ m}^{-1}, \quad (27)$$

$$\alpha = 7.297\,352\,537\,6 \times 10^{-3}. \quad (28)$$

Table II uses experimental values from HyperPhysics, maintained by Georgia State University.³ Moreover, this value corresponds to the value of the latest Japanese handbook.

TABLE II. Comparison of the theoretical and experimental values of the spectral wavelength of a Balmer series.

n	Quantum theoretical value (nm)	Experimental value (nm)	Value predicted by this paper (nm)
3	656.112	656.272	656.125
4	486.009	486.133	486.017
5	433.937	434.047	433.943
6	410.070	410.174	410.076
7	396.9074	397.0072	396.9131
8	388.8073	388.9049	388.8128
9	383.4422	383.5384	383.4476

While the theoretical value and experimental value for the spectral wavelength of a hydrogen atom are generally thought to be in complete agreement, the value predicted by this paper was closer to the experimental value.

Because there is a slight discrepancy between the value calculated in this paper using Eq. (26) and the theoretical and experimental values derived using Rydberg formula (24), this position of this paper is that the conventional view of regarding the Balmer series' spectral theoretical and experimental values should be reconsidered.

(2) A system that performs a simple harmonic oscillation is called a harmonic oscillator. In quantum mechanics, the following energy eigenvalue can be obtained for the energy of such a harmonic oscillator:

$$E_n = \left(n + \frac{1}{2} \right) \hbar \omega, \quad n = 0, 1, 2, \dots \quad (29)$$

Here, n is the number of oscillators, and even for a ground state of $n=0$, this oscillator is known to have a zero-point energy of $\hbar \omega / 2$.

Deriving E_{ng} from Eq. (21d) we obtain $-m_e c^2 / 2$, and this paper considers E_{ng} to be an energy that closely resembles the harmonic oscillator zero-point energy. If we consider E_{ng} to be zero-point energy, it would not be unusual even if H_0 existed; so is there some reason why the existence of a zero-point energy for a hydrogen atom should be prohibited?

IV. CONCLUSION

This paper could not present proof of the existence of H_0 . Therefore, this paper must take the position of (1) or (2) and, for now, we shall support the position of (2).

There is no evidence of the existence of H_0 as considered in this paper of being seriously debated during the formulation of quantum theory. As far as can be determined from papers and writings of the time, physicists appear to have accepted without doubt that the lowest energy of a hydrogen atom is the ground state energy of $n=1$.

Considering these conditions and the discussion of this paper, the assertion of this paper, that the existence of H_0 should be re-examined, is supported.

ACKNOWLEDGMENTS

Appendix A was borrowed and translated from the Japanese language textbook of Dr. H. Ezawa. I wish to express my gratitude to Dr. H. Ezawa.

Also, I created a revised draft in order to reflect the comments of Physics Essays reviewers; in doing so I borrowed the data of Table I from the German language writings related to A. Sommerfeld's "Atomic Structure and Spectral Lines." I would like to express my thanks to the Physics Essays reviewers and to Sommerfeld.

APPENDIX A

Let us review the energy of an electron inside a hydrogen atom. Let us suppose that an atomic nucleus is at rest because it is heavy, and consider the situation where an electron (electric charge $-e$ and mass m_e) orbits at speed v along an orbit (radius r) with the atomic nucleus as its center. An equation describing the motion is as follows:

$$\frac{m_e v^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}. \quad (A1)$$

From this, we obtain

$$\frac{1}{2} m_e v^2 = \frac{1}{2} \frac{1}{4\pi\epsilon_0} \frac{e^2}{r}. \quad (A2)$$

Meanwhile, the potential energy of the electron is

$$V(r) = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r}. \quad (A3)$$

Since the right side of Eq. (A2) is $-1/2$ times the potential energy, Eq. (A2) indicates that

$$V(r) = -2 \left(\frac{1}{2} m_e v^2 \right). \quad (A4)$$

Therefore, the total electron energy

$$E = \frac{1}{2} m_e v^2 + V(r) \quad (A5a)$$

$$= \frac{V(r)}{2} \quad (A5b)$$

$$= -\frac{1}{2} \frac{1}{4\pi\epsilon_0} \frac{e^2}{r}. \quad (A5c)$$

Incidentally, Bohr used the following quantum condition when deriving Eq. (5):

$$p_n \times 2\pi r_n = 2\pi n \hbar, \quad n = 1, 2, \dots \quad (A6)$$

Multiplying both sides of Eq. (A2) by r^2 and using the quantum condition of Eq. (A6), we obtain the following equation:

$$r_n = \frac{4\pi\epsilon_0 \hbar^2 n^2}{m_e e^2} \quad (A7a)$$

$$= a_B n^2, \quad n = 1, 2, \dots \quad (A7b)$$

Here, a_B is the Bohr radius.

If we add the notation n to E and r of Eq. (A5c), we obtain the following:

$$E_n = -\frac{1}{2} \frac{e^2}{4\pi\epsilon_0 r_n}. \quad (\text{A8})$$

Equation (5) is obtained by substituting r_n into Eq. (A7). It is a known fact that the reason for using Eq. (A6) is because it can be used to derive Eq. (5), the energy of a hydrogen atom, and that there is no other basis besides this.

APPENDIX B

When each of the photonic energy and electron's kinetic energy reaches $m_e c^2/2$, the electron cannot obtain more kinetic energy than this, and it is also unable to decrease its potential energy. Thus, the following inequality [Eq. (2)] is true:

$$0 \geq V(r) \geq -m_e c^2.$$

Therefore, there exists a minimum value of potential energy, whereupon the following relationship is established:

$$-\left(\frac{1}{4\pi\epsilon_0}\right) \frac{e^2}{r_e} = -m_e c^2. \quad (\text{B1})$$

The location that satisfies this relationship is the distance of closest approach r_e , which indicates how close the electron comes to the center of the atom. From Eq. (B1), r_e is the following value:

$$r_e = \frac{1}{4\pi\epsilon_0} \frac{e^2}{m_e c^2} \quad (\text{B2a})$$

$$= r_c. \quad (\text{B2b})$$

Here, r_c is the classical electron radius.

¹K. Suto, Phys. Essays **22**, 135 (2009).

²CODATA Internationally recommended values of the Fundamental Physical Constants (2006), <http://physics.nist.gov/cuu/Constants/index.html>.

³C. R. Nave, HyperPhysics: Hydrogen Spectrum, Georgia State University (2008), <http://hyperphysics.phy-astr.gsu.edu/Hbase/hyde.html#c4>.