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Potential Energy of the Electron in a Hydrogen Atom and a Model of a Virtual Particle Pair Constituting the Vacuum

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Abstract

In a previously published paper, the author made some mistakes in calculating the potential energy of the electron in a hydrogen atom. Those mistakes occurred due to applying a potential energy formula with a certain range of application in a region where it is not applicable. Therefore, this paper corrects that error by deriving a formula for potential energy with no range of application. The paper also proposes a model in which a virtual particle pair present in the vacuum region inside a hydrogen atom simultaneously has a photon with positive energy and a photon with negative energy (In this paper, these photons are called dark photons). In the state where the relativistic energy E_{re} is zero, the sum of the positive energy and negative energy of the virtual particle pair becomes zero. According to this model, this makes it possible for the particles to release photons, and capture negative energy.

Keywords: Hydrogen Atom, Potential Energy, Model of a Virtual Particle Pair, Dark Photon, Dark Matter, Proton Radius

1. Introduction

One of the most important relationships in the Special Theory of Relativity (STR) is as follows:

$$\left(m_0 c^2\right)^2 + \boldsymbol{p}^2 c^2 = \left(mc^2\right)^2. \tag{1}$$

Here, mc^2 is the relativistic energy of an object or a particle, and $m_0 c^2$ is the rest mass energy.

Currently, Einstein's relationship (1) is used to describe the energy and momentum of particles in free space, but for explaining the behavior of bound electrons inside atoms, opinion has shifted to quantum mechanics as represented by equations such as the Dirac's relativistic wave equation.

For reasons such as these, there was no search for a relationship between energy and momentum applicable to an electron in the hydrogen atom. However, the author has ventured to take up this problem, and derived the following relationship (Suto, 2011).

$$E_{\mathrm{re},n}^2 + \boldsymbol{p}_n^2 c^2 = \left(m_{\mathrm{e}} c^2\right)^2, \qquad n = 1, 2, \cdots. \tag{2}$$

Here, $E_{\mathrm{re},n}$ is the relativistic energy of the electron, described with an absolute scale. From Equations (1) and (2) it is evident that, if a stationary electron begins to move in free space, or is incorporated into an atom, then the energy which serves as the departure point is the rest mass energy. Consider the case where an electron currently stationary in free space is drawn to a proton to form a hydrogen atom. At this time, the rest mass energy of the electron decreases.

The decrease in rest mass energy of the electron is expressed as $-\Delta m_{\mathrm{e}} c^2$. If the energy of the photon released when an electron is drawn into a hydrogen atom is taken to be $h\nu$, and the kinetic energy acquired by the electron is taken to be K, then the following relationship holds.

$$-\Delta m_{\mathrm{e}} c^2 + h\nu + K = 0. \tag{3}$$

The author presented the following equation as an equation indicating the relationship between the rest mass energy and potential energy of the electron in a hydrogen atom (Suto, 2009).

$$V(r) = -\Delta m_e c^2. \tag{4}$$

From Equations (3) and (4), it is evident that the following relationship holds between potential energy and kinetic energy.

$$\frac{1}{2}V(r) = -K. \tag{5}$$

Also, the potential energy $V(r)$ of the electron is assumed to be 0 when the electron is at rest at a position infinitely far from the proton, and thus it becomes smaller than that inside the atom, and can be described as follows.

$$V(r) = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}. \tag{6}$$

There is a lower limit to potential energy, and the range which energy can assume is as follows.

$$-\Delta m_e c^2 \le V(r) < 0. \tag{7}$$

Here, if $-m_e c^2$ is substituted for V in Equation (6), then the r is

$$r = \frac{e^2}{4\pi\varepsilon_0 m_e c^2} = r_e. \tag{8}$$

Here, r_e is the classical electron radius.

From this, it is evident that Equation (6) has the following range of application.

$$r_e \le r. \tag{9}$$

However, the author also applied Equation (6) in the range where $r < r_e$. Thus, in the following section, a formula for potential energy with no range of application is derived, and that error is corrected.

2. Formula for Potential Energy of the Electron with No Range of Application

The relativistic energy E_{re} of the electron forming a hydrogen atom can be approximately defined as follows.

$$E_{re} = m_e c^2 + E \tag{10a}$$

$$= m_e c^2 + V(r) + K \tag{10b}$$

$$= m_e c^2 + \frac{1}{2}V(r) \tag{10c}$$

$$= m_e c^2 - \frac{1}{2}\Delta m_e c^2. \tag{10d}$$

Equation (10a) is an approximation is because the total mechanical energy E of a hydrogen atom derived by Bohr is an approximate value. (A rigorous definition of $E_{re,n}$ is given below.)

Here, the E in Equation (10a) corresponds to the decrease in the rest mass energy $m_e c^2$ of the electron, and E_{re} corresponds to the remaining part of $m_e c^2$.

The following constraint holds regarding the relativistic energy E_{re} of the electron due to Equation (10d) (here, the discussion is limited to the ordinary energy levels of the electron).

$$\frac{1}{2}m_e c^2 \le E_{re} < m_e c^2. \tag{11}$$

The following formula can also be derived from Equation (10b).

$$V(r) = E_{re} - m_e c^2 - K, \quad K = -E = m_e c^2 - E_{re}. \tag{12}$$

Equation (12) is a formula for potential energy with no range of application. To determine the potential energy in all regions within a hydrogen atom, Equation (6) alone is not sufficient, and the support of Equation (12) is needed.

Incidentally, if Equation (2) is solved for energy, the following solutions can be derived (Suto, 2014a, Suto, 2014b).

$$E_{re,n} = \pm m_e c^2 \left(\frac{n^2}{n^2 + \alpha^2}\right)^{1/2}, \quad n = 0, 1, 2, \cdots. \tag{13}$$

Here, α is the following fine-structure constant.

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c}. \tag{14}$$

If 0 is substituted for n in Equation (13),

$$E_{\mathrm{re},0} = 0. \tag{15}$$

Here, Equation (13) is divided into the following two equations, by taking the positive energy levels among the relativistic energy levels of the electron forming a hydrogen atom to be $E^{+}_{\mathrm{re},n}$, and the negative energy levels to be $E^{-}_{\mathrm{re},n}$.

$$E^{+}_{\mathrm{re},n} = m_{\mathrm{e}} c^2 \left(\frac{n^2}{n^2 + \alpha^2} \right)^{1/2}, \quad n = 1, 2, \cdots. \tag{16}$$

$$E^{-}_{\mathrm{re},n} = -m_{\mathrm{e}} c^2 \left(\frac{n^2}{n^2 + \alpha^2} \right)^{1/2}, \quad n = 1, 2, \cdots. \tag{17}$$

Incidentally, the virtual particle pairs constituting the vacuum are formed from a virtual electron and a virtual positron. As will be discussed below, the energy when $n = 0$ is thought to be not the energy of the electron, but the energy of a virtual electron, and thus it is excluded here. ($E_{\mathrm{re}} = 0$ is the energy of the virtual particle pair. However, the problem being addressed here is the energy of the electron, so here $E_{\mathrm{re},0} = 0$ is regarded as the energy level of the virtual electron).

When Equation (16) is used, the normal energy levels of a hydrogen atom are as follows.

$$E_n = E^{+}_{\mathrm{re},n} - m_{\mathrm{e}} c^2 = m_{\mathrm{e}} c^2 \left[\left(1 + \frac{\alpha^2}{n^2} \right)^{-1/2} - 1 \right], \quad n = 1, 2, \cdots. \tag{18}$$

Now, if a Taylor expansion is performed on the right side of Eq. (18),

$$E_n = m_{\mathrm{e}} c^2 \left[\left(1 + \frac{\alpha^2}{n^2} \right)^{-1/2} - 1 \right] \approx m_{\mathrm{e}} c^2 \left[\left(1 - \frac{\alpha^2}{2n^2} + \frac{3\alpha^4}{8n^4} \right) - 1 \right] \approx m_{\mathrm{e}} c^2 \left(-\frac{\alpha^2}{2n^2} + \frac{3\alpha^4}{8n^4} \right). \tag{19}$$

When this is done, the equations for the energies is as follows.

$$E_n \approx -\frac{\alpha^2}{2n^2} m_{\mathrm{e}} c^2, \quad n = 1, 2, \cdots. \tag{20}$$

Incidentally, in the classical quantum theory of Bohr, the energy levels $E_{\mathrm{B},n}$ of a hydrogen atom are given by the following formula. (Here the B in $E_{\mathrm{B},n}$ stands for “Bohr”).

$$E_{\mathrm{B},n} = -\frac{1}{2} \left(\frac{1}{4\pi\varepsilon_0} \right)^2 \frac{m_{\mathrm{e}} e^4}{\hbar^2} \cdot \frac{1}{n^2} = -\frac{\alpha^2}{2n^2} m_{\mathrm{e}} c^2, \quad n = 1, 2, \cdots. \tag{21}$$

From this, it is evident that Bohr’s energy equation, Equation (21), is an approximation of Equation (18).

The following compares energies when $n = 1$.

Value predicated by this paper Equation (18): $E_1 = -13.60515 \text{ eV}$. (22a)

Value predicted by Bohr Equation (21): $E_{\mathrm{B},1} = -13.60569 \text{ eV}$. (22b)

$$\frac{E_{\mathrm{B},1}}{E_1} = 1.0000397. \tag{22c}$$

In Equation (10a), E_{re} was defined from $m_{\mathrm{e}} c^2$ and E, but it is actually correct to define E from $m_{\mathrm{e}} c^2$ and E_{re}. Incidentally, the relativistic energy of the electron can also be written as follows.

$$E_{\mathrm{re},n} = m_{\mathrm{e}}c^2 - \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n} \tag{23a}$$

$$= m_{\mathrm{e}}c^2\left(1 - \frac{r_{\mathrm{e}}/2}{r_n}\right). \tag{23b}$$

Next, Equations (16) and (23b) are joined with an equals sign. That is,

$$\frac{n^2}{n^2+\alpha^2} = \left(\frac{r_n - r_{\mathrm{e}}/2}{r_n}\right)^2. \tag{24}$$

From this, the following quadratic equation is obtained.

$$r_n^2 - \left(\frac{n^2+\alpha^2}{\alpha^2}\right)r_{\mathrm{e}}r_n + \left(\frac{n^2+\alpha^2}{\alpha^2}\right)\frac{r_{\mathrm{e}}^2}{4} = 0. \tag{25}$$

If this equation is solved for r_n,

$$r_n = \frac{r_{\mathrm{e}}}{2}\left(1+\frac{n^2}{\alpha^2}\right)\left[1 \pm \left(1+\frac{\alpha^2}{n^2}\right)^{-1/2}\right] \tag{26}$$

When the Taylor expansion of Equation (26) is taken, the result is as follows.

$$r_n \approx \frac{r_{\mathrm{e}}}{2}\left(1+\frac{n^2}{\alpha^2}\right)\left[1 \pm \left(1-\frac{\alpha^2}{2n^2}+\frac{3\alpha^4}{8n^4}\right)\right] \tag{27}$$

Here, the negative solution r_n^- of Equation (27),

$$r_n^- \approx \frac{r_{\mathrm{e}}}{4} + \frac{\alpha^2 r_{\mathrm{e}}}{16n^2} = \frac{r_{\mathrm{e}}}{4} + \frac{a_{\mathrm{B}}}{n^2}\left(\frac{\alpha}{2}\right)^4. \tag{28}$$

Since r^- converges to $r_{\mathrm{e}}/4$, $r_{\mathrm{e}}/4$ can be regarded as the radius of the atomic nucleus of a hydrogen atom (i.e., the proton). Here, the theoretical value of the proton radius is:

$$r_\infty^- = \frac{r_{\mathrm{e}}}{4} = 0.704485080675\times10^{-15}\ \mathrm{m}. \tag{29}$$

However, if an attempt is actually made to measure the size of the proton (atomic nucleus), the energy of the proton changes. The size of the proton depends on the proton's energy, and thus the measured value does not match with Equation (29) (Randolf, 2010; Suto, 2014c). In addition, it is possible to predict that a different measurement value will be obtained from an experiment using a different measurement method.

Next, when the value obtained by setting $n=0$ in Equation (26) is taken to be r_0,

$$r_0 = \frac{r_{\mathrm{e}}}{2}. \tag{30}$$

Here, State 0 with $n=0$ is defined as follows.

$$\text{State 0:} \quad r = \frac{r_{\mathrm{e}}}{2}, \quad E_{\mathrm{re}} = 0. \tag{31}$$

3. Correction of Potential Energy of the Electron in a Hydrogen Atom

The points where the author made a mistake in the value of potential energy of the electron are $*1$ to $*3$ in the following diagram (see Figure 1) (Suto, 2017).

Originally, the potential energy at $*1$ to $*3$ was found from Equation (6), but potential energy in this region must be found from Equation (12). That is,

$$*1\ V(r) = -2m_{\mathrm{e}}c^2 \rightarrow -m_{\mathrm{e}}c^2. \tag{32a}$$

$$*2\ V(r) = -3m_{\mathrm{e}}c^2 \rightarrow -m_{\mathrm{e}}c^2. \tag{32b}$$

$$*3\ V(r) = -4m_{\mathrm{e}}c^2 \rightarrow -2m_{\mathrm{e}}c^2. \tag{32c}$$

Next, the regions in a hydrogen atom are classified as follows at the level of classical theory while taking into account Equations (7) to (12) (see Table).

Table 1. Regions and states. This is Figure 1 made into a table. Here, the value of K was found from Equation (12)

Regions and states	r	E_{re}	$V(r)$	K
Region A	$r_{\mathrm{e}} < r_n^+$	$\frac{m_{\mathrm{e}}c^2}{2} < E_{\mathrm{re},n}^+ < m_{\mathrm{e}}c^2$	$-m_{\mathrm{e}}c^2 < V(r) < 0$	$\frac{m_{\mathrm{e}}c^2}{2} > K > 0$
State a	r_{e}	$\frac{m_{\mathrm{e}}c^2}{2}$	$-m_{\mathrm{e}}c^2$	$\frac{m_{\mathrm{e}}c^2}{2}$
Region B	$\frac{r_{\mathrm{e}}}{2} < r^+ < r_{\mathrm{e}}$	$0 < E_{\mathrm{re}}^+ < \frac{m_{\mathrm{e}}c^2}{2}$	$-m_{\mathrm{e}}c^2$	$0 < K < \frac{m_{\mathrm{e}}c^2}{2}$
State 0	$\frac{r_{\mathrm{e}}}{2}$	0	$-m_{\mathrm{e}}c^2$	0
Region C	$\frac{r_{\mathrm{e}}}{3} < r^- < \frac{r_{\mathrm{e}}}{2}$	$-\frac{m_{\mathrm{e}}c^2}{2} < E_{\mathrm{re}}^- < 0$	$-m_{\mathrm{e}}c^2$	$-\frac{m_{\mathrm{e}}c^2}{2} < K < 0$
State c	$\frac{r_{\mathrm{e}}}{3}$	$-\frac{m_{\mathrm{e}}c^2}{2}$	$-m_{\mathrm{e}}c^2$	$-\frac{m_{\mathrm{e}}c^2}{2}$
Region D	$\frac{r_{\mathrm{e}}}{4} < r_n^- < \frac{r_{\mathrm{e}}}{3}$	$-m_{\mathrm{e}}c^2 < E_{\mathrm{re},n}^- < -\frac{m_{\mathrm{e}}c^2}{2}$	$-2m_{\mathrm{e}}c^2 < V(r) < -m_{\mathrm{e}}c^2$	$0 > K > -\frac{m_{\mathrm{e}}c^2}{2}$
State d	$\frac{r_{\mathrm{e}}}{4}$	$-m_{\mathrm{e}}c^2$	$-2m_{\mathrm{e}}c^2$	0

Region A is the region where the electron forming a hydrogen atom exists. However, in Region B, there is no change in the potential energy of the particle. Therefore, what exists in this region is not charged particles. Thus, this paper predicts that Region B is a region of a virtual particle pair formed from a virtual electron and virtual positron. Virtual particle pairs are the particles constituting the vacuum. In this region, the kinetic energy of a virtual particle pair decreases as the particle pair approaches the atomic nucleus. However, in the regions of the electron where $r_{\mathrm{e}} < r_n^+$ and $r_{\mathrm{e}}/4 < r_n^- < r_{\mathrm{e}}/3$, kinetic energy increases as the electron approaches the atomic nucleus.

A virtual particle pair with $E_{\mathrm{re}} = 0$ exists in State 0. When this virtual particle pair absorbs $m_{\mathrm{e}}c^2$ of energy, the virtual particle pair transitions into State a. (At this time, the energy of the virtual electron is 1/2 the energy of the virtual particle pair, and therefore is $m_{\mathrm{e}}c^2/2$).

Also, this paper predicts that this virtual particle pair will separate into a virtual electron and virtual positron in State a.

Region C is a region symmetrical with Region B in terms of energy. The virtual particle pairs existing in this region have a negative energy (mass).

Region D is symmetric with Region A in terms of energy. Electrons in this region have negative energy (mass) in Equation (17). The author has already pointed out that the system formed from a proton and an electron with negative energy is a candidate for dark matter, a type of matter whose true nature is unknown. (The author calls electrons with this negative energy dark electrons, and photons with negative energy dark photons.)

When Figure 1 is corrected based on the above, the result is as follows (see Figure 2).

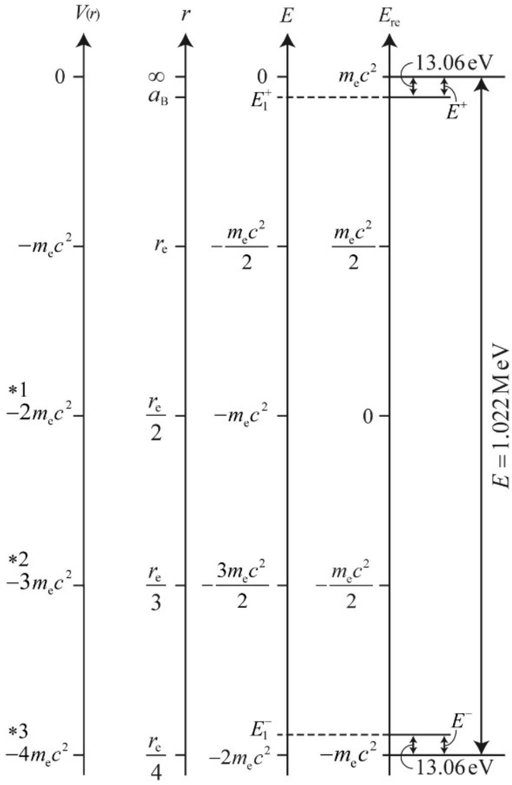

Figure 1. Relationship between energies of the electron and virtual electron present in a hydrogen atom, and their positions r. The region where the electron forming the hydrogen atom exists is $r_e < r$. $r_e/3 < r < r_e$ is the region of the virtual particle pair constituting the vacuum, but the energy in this diagram indicates the energy of the virtual electron (The energy of the virtual particle pair is twice the energy of the virtual electron). Also, $r_e/4 < r < r_e/3$ is the region of the electron with negative energy (mass). This diagram is cited from another paper, but the values for potential energy at $*1$ to $*3$ are mistaken, and thus they will be corrected in this paper

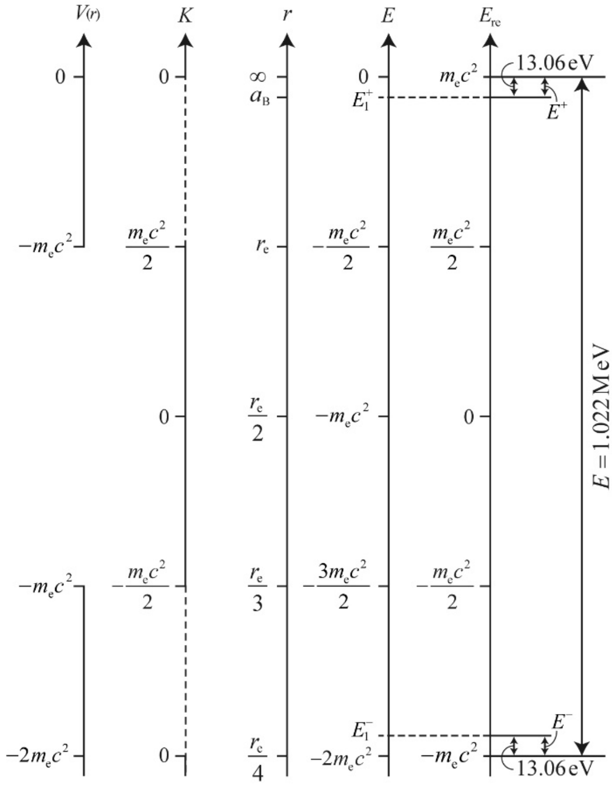

Figure 2. In this figure, potential energy (vertical line) has been erased in the region where potential energy does not exist ($r_e/3 < r < r_e$). Also, as the electron in Region A, and the dark electron in Region D, approach the atomic nucleus, the kinetic energy of the electron increases. Thus K in this region is shown with a dashed line

4. Discussion

In the previous section, the potential energy value of the electron was corrected, and thus the original purpose of this paper was achieved. However, there are still a number of points that can be discussed.

1) How does a virtual particle pair with $E_{re} = 0$ acquire negative energy? This paper examines two interpretations.

Interpretation 1: A virtual particle pair absorbs a dark photon with negative energy, and lowers its energy level.

However, a dark photon has never been observed in the natural world. Therefore, this interpretation cannot be supported. Thus, the previous view of virtual particle pairs with $E_{re} = 0$ is reexamined.

That is,

Previous view: A virtual particle pair with $E_{re} = 0$ is in a state where rest mass energy has been completely consumed, i.e., (to use a vulgar expression) a naked state unclothed by photon energy.

The interpretation of this paper (Interpretation 2): A virtual particle with $E_{re} = 0$ simultaneously has a photon with positive energy and a dark photon with negative energy. If here the positive photon energy is taken to be E_P ($0 < E_P$) and the energy of the dark photon is taken to be E_{DP} ($E_{DP} < 0$), then $E_{re} = 0$ can be defined as the state where the sum of E_P and E_{DP} is zero. That is,

$$E_{\mathrm{re},0} = E_{\mathrm{P}} + E_{\mathrm{DP}} = 0, \quad E_{\mathrm{P}} = \left|E_{\mathrm{DP}}\right| \neq 0. \tag{33}$$

Incidentally, the definition of the rest mass energy E_0 of the electron is $E_0 = m_e c^2$. However, if the model here is used, this energy can be defined as follows.

$$E_0 = E_{\mathrm{P}} + E_{\mathrm{DP}} = E_{\mathrm{P}} - \left|E_{\mathrm{DP}}\right| = m_e c^2. \tag{34}$$

This paper cannot predict the energies of the photon and dark photon belonging to the virtual particle pair with $E_{\mathrm{re}} = 0$. However, for a dark electron to attain State d, the virtual particle pair with $E_{\mathrm{re}} = 0$ must simultaneously have a $m_e c^2$ photon and a $-m_e c^2$ dark photon.

Also, according to this model of the virtual particle pair, the $E^{-}_{\mathrm{re},n}$ in Equation (17) is not the energy of the dark photon belonging to the dark electron. $E^{-}_{\mathrm{re},n}$ corresponds to the sum of the energy of the photon belonging to the dark electron, and the energy of the dark photon. That is,

$$E^{-}_{\mathrm{re},n} = E_{\mathrm{P}} + E_{\mathrm{DP}} = E_{\mathrm{P}} - \left|E_{\mathrm{DP}}\right|, \quad E_{\mathrm{P}} < \left|E_{\mathrm{DP}}\right|. \tag{35}$$

2) To estimate the number of virtual particle pairs present in the vacuum region inside a hydrogen atom, let us look at triplet production.

Now, consider the case where an incident *γ*-ray has the energy corresponding to the mass of 4 electrons (2.044 MeV). If this is discussed classically, the *γ*-ray can create an electron and positron near $r = r_e / 2$ (see Figure 3).

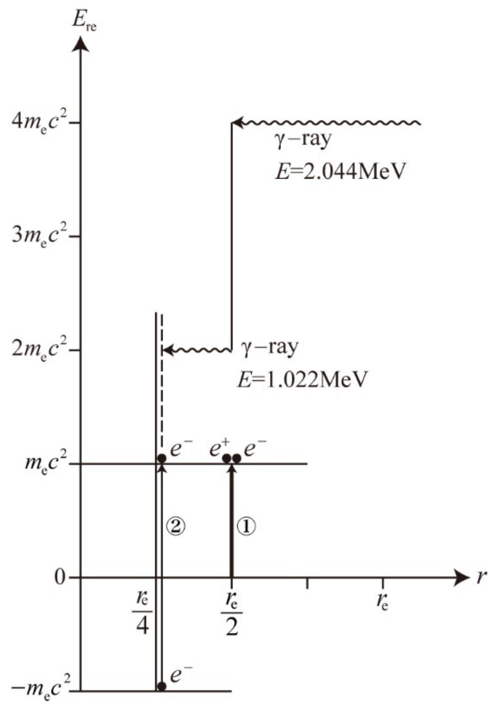

Figure 3. Interpretation of this paper regarding triplet production

This *γ*-ray will give 1.022 MeV of energy to the virtual particles at $r = r_e / 2$, and an electron-positron pair will be created (↑①). When this *γ*-ray approaches closer to the atomic nuclear, and the electron in the orbital around the proton absorbs this energy, the electron will be excited and appear in free space (↑②). If multiple virtual particle pairs exist in the $E_{\mathrm{re}} = 0$ state, then there is potentially a probability that two electrons and two positrons are produced in the process in ①. However, a phenomenon of this sort has not been observed.

Even if 1.022 MeV of energy is consumed in this pair creation, the *γ*-ray still has the energy of corresponding to the mass of 2 electrons (1.022 MeV). If the *γ*-ray gives energy to an electron in the orbital near the proton, the electron will be excited and appear in free space. As a result, 2 electrons and 1 positron will appear in free space.

However, if multiple virtual particle pairs exists in the $E_{\mathrm{re},0}$ state, then in process ①, there should also be a probability of producing two pairs (2 electrons and 2 positrons) from an energy of 2.044 MeV ($4m_{\mathrm{e}}c^2$). However, quadruplet production has never been actually observed. Thus, this paper predicts 1 virtual particle pair in State 0. There is also a probability that, aside from an electron, a single virtual electron and virtual positron are present in Region A. Taking these points into consideration, there is a possibility that the previous definition of the hydrogen atom is too simple, and reconsideration may be necessary. That is,

Previous view: Hydrogen atom = 1 proton + 1 electron

Model to be examined: Hydrogen atom = 1 proton + 1 electron + 1 virtual particle pair (or 1 virtual electron + 1 virtual positron)

Here, if one virtual particle pair is added, then the model is applied when the particle pair is present in Region B, and if 2 virtual particles are added, the model is applied when the virtual particles are present in Region A.

3) If the energy absorbed by the virtual particle pair with $E_{\mathrm{re}} = 0$ is in the range $m_{\mathrm{e}}c^2 < E < 2m_{\mathrm{e}}c^2$, then the virtual electron and virtual positron separated in State a are present temporarily in Range A. Now, what is the difference between the electron forming the hydrogen atom and the virtual electron? It is difficult to discriminate these 2 particles from the perspective of energy. However, the virtual electron and virtual positron in Region A are likely not completely separated, and in a state of quantum entanglement. Therefore, the electron and virtual electron in Region A are not in the same state.

There is also thought to be a probability that a virtual positron separated from a virtual electron at $r = r_{\mathrm{e}}$ will approach the electron of the hydrogen atom and form a virtual particle pair. If a new virtual particle pair is formed here, the remaining virtual electron will then behave as the electron of the hydrogen atom. If this model is assumed to be correct, the electron in the hydrogen atom does not describe a continuous trajectory, and its motion is discontinuous. Also, it is predicted that the electron will behave as though it had moved to another location instantaneously (at super luminal speed).

4) As is also evident from Figure 2, the position occupied by the electron and dark electron in the hydrogen atom, and the region of energy, are only a small part of the whole. The remaining majority is the region of the virtual particle pair and virtual particles (virtual electron and virtual positron). If even the virtual particle pair is included in the constituents of the hydrogen atom, then there will be a need to derive the energy levels of the virtual particle pair. The energy levels of the electron and dark electron are discrete, and thus based on common sense, the energy levels of the virtual particle pair are also predicted to be discrete.

5. Conclusion

1) In this paper, Equation (12) was used to correct the value for potential energy in a hydrogen atom, previously found incorrectly by the author. As a result, Figure 1 has been corrected as shown in Figure 2.

2) According to the model proposed in this paper, a virtual particle pair simultaneously has a photon with positive energy E_{P} and a dark photon with negative energy E_{DP}. In this case, the previous energy is redefined as follows.

i) If the relativistic energy of a virtual particle pair is zero,

Previous definition: $$E_{\mathrm{re},0} = 0. \tag{36}$$

Definition in this paper: $$E_{\mathrm{re},0} = E_{\mathrm{P}} + E_{\mathrm{DP}} = 0, \quad E_{\mathrm{P}} = |E_{\mathrm{DP}}| \neq 0. \tag{37}$$

ii) Rest mass energy of an electron E_0

Previous definition: $$E_0 = m_{\mathrm{e}}c^2. \tag{38}$$

Definition in this paper: $$E_0 = E_{\mathrm{P}} + E_{\mathrm{DP}} = E_{\mathrm{P}} - |E_{\mathrm{DP}}| = m_{\mathrm{e}}c^2. \tag{39}$$

iii) Energy levels of a dark electron with negative energy

Definition in this paper: $$E_{\mathrm{re},n}^{-} = E_{\mathrm{P}} + E_{\mathrm{DP}} = E_{\mathrm{P}} - |E_{\mathrm{DP}}|, \quad E_{\mathrm{P}} < |E_{\mathrm{DP}}|. \tag{40}$$

Incidentally, the existence of dark photons cannot be directly demonstrated by experiment, just like virtual particle pairs. However, if in the future it is possible to demonstrate the existence of negative energy levels $E_{\mathrm{re},n}^{-}$ and dark electrons in the hydrogen atom, then the existence of dark photons will also be simultaneously demonstrated.

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