True factors determining the ratio of space contraction and time dilation predicted by the special theory of relativity

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Abstract: According to the special theory of relativity (STR), when the velocity of a moving body increases, the length of the body contracts in the direction of motion, and the passage of time slows down in its coordinate system. This conclusion can be derived from the equations of the Lorentz transformation. Inside the atom, however, potential energy must be taken into account, and the equations of the Lorentz transformation cannot be applied. In situations where the transformation equations newly derived by the author can be applied, if the velocity of a moving body increases, then the body elongates in the direction of motion, and the passage of time speeds up in its coordinate system. At the same time, the relativistic energy and mass of the body decrease. In the special theory of relativity, it is the relative velocity between the stationary frame and moving frame which determines length in the direction of motion of the moving body, and the tempo of the time which passes in the body's coordinate system. However, this paper rejects the claim that relativistic velocity is the factor which determines the body's length and the time which passes. Instead it concludes that the true cause is the ratio of the body's rest mass energy and relativistic energy—a ratio which varies as the velocity of the moving body increases and decreases. © *2014 Physics Essays Publication*. [http://dx.doi.org/10.4006/0836-1398-27.4.580]

Résumé: Selon la théorie de la relativité restreinte, si la vitesse d'un corps en mouvement augmente, la longueur de ce corps dans la direction du mouvement rétrécit et le temps qui s'écoule dans ce système inertiel ralentit. Ceci est la conclusion à laquelle aboutit la formule de transformation de Lorentz. Cependant, la formule de transformation de Lorentz ne peut pas s'appliquer dans un atome dont on doit considérer l'énergie potentielle. Dans la situation vers laquelle nous guide de manière nouvelle l'auteur et à laquelle on peut appliquer la formule de transformation, si la vitesse d'un corps en mouvement augmente, la longueur du corps dans la direction du mouvement s'étend et le temps qui s'écoule dans ce système inertiel se poursuit. De plus, la masse et l'énergie relative de ce corps diminuent simultanément. Dans la théorie de la relativité restreinte, c'est la vitesse relative entre le système au repos et le système en mouvement qui détermine la longueur d'un corps dans la direction du mouvement et le rythme du temps dans le système inertiel de ce corps. Néanmoins, ce texte rejette la thèse selon laquelle le facteur déterminant la longueur du corps et le temps qui s'écoule est la vitesse relative; sa conclusion est que le vrai facteur est la proportion de l'énergie de la masse au repos du corps et l'énergie relative (du corps) évoluant selon l'augmentation et la diminution de la vitesse du corps en mouvement.

Key words: Special Theory of Relativity; Einstein's Energy–Momentum Relationship; Lorentz Transformation; Rest Mass Energy; Relativistic Energy.

I. INTRODUCTION

Among the equations which represent the special theory of relativity (STR) are the following Lorentz transformation (1) and Einstein's energy–momentum relationship (2):

$$\begin{aligned} &x = \gamma(x' + vt'), \quad x' = \gamma(x - vt), \\ &y = y', \quad y' = y, \\ &z = z', \quad z' = z, \\ &t = \gamma\left(t' + vx'/c^2\right), \quad t' = \gamma\left(t - vx/c^2\right). \end{aligned} \tag{1}$$

Here, $\gamma = (1 - v^2/c^2)^{-1/2}$, v is the velocity of an inertial system S' as measured in inertial system S.

$$E^2 = \boldsymbol{p}^2c^2 + E_0^2. \tag{2}$$

Here, E_0 is the rest mass energy of the body, and E is the relativistic energy of the body.

According to STR, when the speed of a moving body increases, its length in the direction of motion contracts, and the time elapsed in its inertial system slows down. At the same time, the energy and mass of the body increase.

The equations of STR hold in macrolevel free space, and the following energy–momentum relationship of a bound electron in a hydrogen atom holds true:[1]

$$E_{\mathrm{re},n}^2 + \boldsymbol{p}_n^2c^2 = \left(m_\mathrm{e}c^2\right)^2, \quad n = 1, 2, \cdots. \tag{3}$$

Here, $m_\mathrm{e}c^2$ indicates the rest mass energy of the electron and $E_{\mathrm{re},n}$ indicates the relativistic energy of the electron

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(n is the principal quantum number). $E_{\mathrm{re},n}$ is defined as follows:

$$E_{\mathrm{re},n} = m_{\mathrm{e}}c^2 + E_n, \quad E_n < 0. \tag{4}$$

Here, E_n is the energy of the hydrogen atom given by quantum mechanics.

$E_{\mathrm{re},n}$ is introduced to physics without any special attention. In the textbook of Schiff, the following equation is given for $E_{\mathrm{re},n}$:[2]

$$E = m_{\mathrm{e}}c^2\left[1 - \frac{\gamma^2}{2n^2} - \frac{\gamma^4}{2n^4}\left(\frac{n}{|k|} - \frac{3}{4}\right)\right]. \tag{5}$$

If Eq. (3) is quantized, an equation equivalent to the Dirac's relativistic wave equation can be easily found,[3] and the size of the proton can also be predicted from Eq. (3).[4] Results in author's previous papers have cleared up doubts about Eq. (3).

Now, if the velocity of a particle is set to 0 in Eq. (2), then

$$m = \frac{E}{c^2}. \tag{6}$$

Also, in Newtonian mechanics,

$$m = \frac{p}{v}. \tag{7}$$

From these two equations we obtain,

$$cp = \frac{Ev}{c}. \tag{8}$$

Next, if we substitute this pc in Eq. (2) and rearrange

$$E = \frac{E_0}{\sqrt{1 - v^2/c^2}}. \tag{9}$$

Here, taking into account the relationship in Eq. (6)

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}. \tag{10}$$

In STR, the relativistic energy E of a body becomes larger than the rest mass energy E_0 of the body. Similarly, the relativistic mass m of a body becomes larger than the rest mass m_0 of the body.

If the logic used to derive Eqs. (9) and (10) from Eq. (2) is now applied to Eq. (3), the following two equations can be derived:

$$E_{\mathrm{re},n} = \frac{m_{\mathrm{e}}c^2}{\sqrt{1 + v^2/c^2}}. \tag{11}$$

$$m_{\mathrm{re},n} = \frac{m_{\mathrm{e}}}{\sqrt{1 + v^2/c^2}}. \tag{12}$$

According to Eq. (12), when the velocity of an electron in a hydrogen atom increases, its mass decreases, thereby making acceleration easier.

Now, let us consider two inertial systems S and S' which are moving at a constant speed relative to each other in free space (In the following, inertial system S with be abbreviated as S, and inertial system S' as S').

Let us assume that the distance between two points x_1 and x_2 on the x-axis in S has been measured as L_0 by an observer in S. Next, let us assume that the distance between these two points is measured as L by an observer in S' moving at a constant speed v relative to S. At this time, the observer in S' finds the following relationship between L_0 and L:

$$L = L_0/\gamma = L_0\sqrt{1 - v^2/c^2}. \tag{13}$$

The length of the moving body contracts in the direction of motion. However, according to STR, S and S' are physically equivalent. Therefore, conversely, if an observer in S' measures the distance between two points on the x'-axis in S' as L_0, then the observer in S measures this distance as L.

Also, with regard to time, if a time τ elapses in S' when a time τ_0 elapses in S, then an observer in S' finds the following relationship between τ_0 and τ:

$$\tau = \gamma\tau_0 = \frac{\tau_0}{\sqrt{1 - v^2/c^2}}, \quad \tau_0 < \tau. \tag{14}$$

The time that elapses on a moving clock is slowed down. Also, if an observer in S measures the passage of time in S', then the same effect of slowing down will be found.

Although Eq. (1) is applicable in macrolevel free space where Eq. (2) holds, there is no transformation corresponding to Eq. (1) in the space in the atom where Eq. (3) holds. If symmetry is taken into account, then a transformation may exist which is applicable in the atom.

Deriving that transformation is not so difficult mathematically, but physically we encounter a major problem.

In quantum mechanics, the motion of the electron in the hydrogen atom cannot be conceived as a classical motion, but is really a standing wave ψ.

Drawing some kind of picture of the motion of the electron in the atom is a foolhardy act which ignores quantum mechanics.

However, it would not have been strange even if the transformation derived in this paper were discovered in the early 20th century. Therefore, the author feels that it is permissible for this paper to derive the transformation based on a natural view of classical quantum theory.

II. DERIVATION OF A TRANSFORMATION DIFFERENT FROM THE LORENTZ TRANSFORMATION

In this section, let us try to derive the unknown transformation by referring to the logic used to derive the Lorentz transformation.[5]

Let the system of the atomic nucleus be S, and let S' be the system of the electron which moves at a constant speed in the atom.

The electron is engaged in uniform motion in the atom when the electron is engaged in circular motion. However, in this case, S and S' cannot be regarded as equivalent coordinate systems.

In contrast, if the principle quantum number takes its maximum value, and the electron moves in a highly flattened elliptical orbital, then the electron can be regarded as approximately engaged in uniform motion, provided it is limited to a minute interval of time.

However, if the reader cannot accept the model established by the author, it is fine to assume the coordinate system S' of a particle which cuts across the space in the atom at a constant speed.

This paper postpones discussion of the limits of application of the unknown transformation and places priority on first deriving the transformation.

Now, in quantum mechanics, complex numbers play an essential role, and thus it is assumed that complex numbers will also play an important role when deriving the new transformation. It is also assumed that the new transformation will be linear.

First, taking into account the symmetry which comes from the principle of relativity, assume that the following relations hold:

$$x = iax' + ibt', \tag{15a}$$

$$x' = iax - ibt. \tag{15b}$$

Motion of the origin of S viewed from S' can be found if we let $x = 0$ in the first equation. Conversely, motion of the origin of S' viewed from S can be found if we let $x' = 0$ in the second equation. Those velocities v face in the opposite direction, but have the same magnitude.

As a result, the following condition holds:

$$\frac{b}{a} = v. \tag{16}$$

Next, let us consider the case where a light signal moving in the positive direction on the x-axis is viewed from S and S'. We assume that when the origins of the two inertial systems coincide, propagation of the light signal emitted from the origin of the x-axis can be described using the following equations, from the perspective of S and S', respectively,

$$x = ict, \tag{17a}$$

$$x' = ict'. \tag{17b}$$

If the x and x' in both equations are substituted into Eqs. (15), then the following equations are obtained:

$$ct = (iac + b)t', \tag{18a}$$

$$ct' = (iac - b)t. \tag{18b}$$

If t and t' are deleted in these two equations, and the relationship in Eq. (16) is used, the result is as follows:

$$c^2 = -a^2\left(c^2 + v^2\right). \tag{19}$$

As a result,

$$a = \pm \frac{i}{\sqrt{1 + v^2/c^2}}. \tag{20}$$

Here, we define the following γ_a:

$$\gamma_a = \frac{1}{\sqrt{1 + v^2/c^2}}. \tag{21}$$

When that is done, Eq. (20) becomes as follows:

$$a = \pm i\gamma_a. \tag{22}$$

If the right side of this equation is substituted for a in Eq. (15), then

$$x = \mp \frac{x' + vt'}{\sqrt{1 + v^2/c^2}} = \mp\gamma_a(x' + vt'), \tag{23a}$$

$$x' = \mp \frac{x - vt}{\sqrt{1 + v^2/c^2}} = \mp\gamma_a(x - vt). \tag{23b}$$

For the coefficient in Eq. (1), $\gamma(v) > 1$, but for the coefficient in Eq. (23), $\gamma_a(v) < 1$.

Incidentally, the energy ranges of the electron where Eq. (23) is applicable are likely to be $m_e c^2/2 < E_{re} < m_e c^2$ and $-m_e c^2 < E_{re} < -m_e c^2/2$.

Thus, this paper assumes that the transformation which is applicable in the former energy range is the equation where the coefficient is $+\gamma_a$ which is less strange.

When Eq. (23) is obtained, the equations for time t and t' can be obtained through elementary mathematical calculation.

First, the following t is obtained when the right side of Eq. (17a) is substituted for the x in Eq. (23a):

$$t = \frac{\gamma_a}{ic}(x' + vt'). \tag{24}$$

Next, Eq. (24) becomes as follows when the right side of Eq. (17b) is substituted for x', and the value found from Eq. (17b) is substituted for t' in Eq. (24):

$$t = \gamma_a\left(t' - vx'/c^2\right). \tag{25}$$

Also, the following equation is obtained when t' is derived using the same method:

$$t' = \gamma_a\left(t + vx/c^2\right). \tag{26}$$

In this paper, the transformation indicated below shall be tentatively called the Lorentz transformation II

$$\begin{aligned} &x = \gamma_a(x' + vt'), \quad x' = \gamma_a(x - vt), \\ &y = y', \quad y' = y, \\ &z = z', \quad z' = z, \\ &t = \gamma_a\left(t' - vx'/c^2\right), \quad t' = \gamma_a\left(t + vx/c^2\right). \end{aligned} \tag{27}$$

In addition, if E_n in Eq. (4) is used in place of $E_{re,n}$ then the energy range where Eq. (27) is applicable is $-m_e c^2/2 < E_n < 0$. (However, the energy lower limit was predicted based on the author's paper.)[6]

On the other hand, the energy range of bodies to which the Lorentz transformation is applicable is $m_0 c^2 < E$.

Incidentally, there was no legitimate reason to use the assumptions in Eqs. (15) and (17) used in this paper. Equation (27) was simply taken to be the equation to be eventually derived, and the assumptions were used to make that possible. However, assumptions similar to these assumptions have already been used in STR to derive Eq. (1). Therefore, it was decided that there would be no problem with assuming Eqs. (15) and (17) in this paper. However, this does not mean that Eq. (27) has been determined to have physical meaning.

Now, Eqs. (13) and (14) were derived from the Lorentz transformation (1), and thus in this paper the equations corresponding to Eqs. (13) and (14) shall be derived from Eq. (27).[7]

Define the length L_0 of a body in S' as follows:

$$L_0 = x'_2 - x'_1. \tag{28}$$

If the length of this body is measured from S, the length L is

$$L = x_2 - x_1. \tag{29}$$

If Eq. (27) is used here

$$\begin{aligned} x'_1 &= \gamma_a(x_1 - vt),\\ x'_2 &= \gamma_a(x_2 - vt). \end{aligned} \tag{30}$$

Therefore,

$$x'_2 - x'_1 = \gamma_a(x_2 - x_1), \tag{31}$$

$$L = L_0/\gamma_a = L_0\sqrt{1 + v^2/c^2}. \tag{32}$$

If the length of a body in S' is measured from S, the body elongates in the direction of motion. This result differs from the prediction of STR, but the reason for that is the difference in the coefficients γ and γ_a.

Next, let us examine time.[8]

Assume a single clock is at rest at the point $x' = x'_0$ in S'. Let us consider two events which occur at different times.

Event 1: (x'_0, t'_1)

Event 2: (x'_0, t'_2)

Let us calculate the time coordinate of the two events measured in S, which is moving at a constant velocity v relative to S'.

The following equations are obtained using Eq. (25):

$$t_1 = \gamma_a\left(t'_1 - vx'_0/c^2\right), \tag{33a}$$

$$t_2 = \gamma_a\left(t'_2 - vx'_0/c^2\right). \tag{33b}$$

From this,

$$t_2 - t_1 = \gamma_a\left(t'_2 - t'_1\right). \tag{34}$$

If the time difference t_2–t_1 is written as τ and time difference t'_2–t'_1 is as τ_0, then the following equations are obtained for an observer in S:

$$\tau = \gamma_a\tau_0 = \frac{\tau_0}{\sqrt{1 + v^2/c^2}}, \quad \tau < \tau_0. \tag{35}$$

When τ_0 elapses in S', then an observer in S concludes that τ elapses in S.

If the time that elapses on a clock in S' is measured from S, then the clock in S' is sped up.

Table I can be obtained from the discussion of this paper.

III. DISCUSSION

If Eqs. (9) and (10) are multiplied with Eq. (13), the result is as follows [here, Eq. (13) is used as the equation when measuring physical quantities in S' from S]:

$$EL = E_0L_0, \tag{36}$$

$$mL = m_0L_0. \tag{37}$$

Also, if Eq. (13) is taken into account, these relations become

$$L = \frac{E_0}{E}L_0 = \frac{m_0}{m}L_0 = \frac{L_0}{\gamma}, \quad \gamma = \frac{E}{E_0} = \frac{m}{m_0} > 1. \tag{38}$$

Now, let us assume that the relativistic energy E of a body moving in free space at the macrolevel is E/E_0 times the rest mass energy E_0. In this situation, when an observer in S measures the length of a ruler in S', the ruler is contracted by E_0/E times in the direction of motion (however, $E_0 < E$).

TABLE I. Comparison of equations which hold in the hydrogen atom with STR equations which hold in macrolevel space. (However, equations marked with an asterisk are not settled equations.)

	Macrolevel free space	Space in the hydrogen atom
Energy–momentum relationship	$E^2 = \boldsymbol{p}^2c^2 + (m_0c^2)^2$	$E^2_{re,n} + \boldsymbol{p}^2c^2_n = (m_ec^2)^2$
Energy range where relation is applicable	$m_0c^2 < E$	$m_ec^2/2 < E_{re} < m_ec^2$
Equation for relativistic energy	$E = \dfrac{m_0c^2}{\sqrt{1 - v^2/c^2}}$	$E_{re} = \dfrac{m_ec^2}{\sqrt{1 + v^2/c^2}}$
Applicable transformation	Lorentz transformation, Eq. (1)	Lorentz transformation II, Eq. (27)*
Length of moving body L predicted by STR	$L = L_0\sqrt{1 - v^2/c^2}$	$L = L_0\sqrt{1 + v^2/c^2}*$
Length of moving body L derived in this paper	$L = \dfrac{E_0}{E}L_0$	$L = \dfrac{E_0}{E_{re}}L_0*$
Time elapsed on moving clock τ_0 predicted by STR	$\tau_0 = \tau\sqrt{1 - v^2/c^2}$	$\tau_0 = \tau\sqrt{1 + v^2/c^2}*$
Time elapsed on moving clock τ_0 derived in this paper	$\tau_0 = \dfrac{E_0}{E}\tau$	$\tau_0 = \dfrac{E_0}{E_{re}}\tau*$
Superluminal velocity	Impossible	Possible

With regard to time, the following equations can be obtained if Eqs. (9) and (10) are multiplied with Eq. (14):

$$E\tau_0 = E_0\tau, \tag{39}$$

$$m\tau_0 = m_0\tau. \tag{40}$$

Also, if Eq. (14) is taken into account, these relations become

$$\tau = \frac{E}{E_0}\tau_0 = \frac{m}{m_0}\tau_0 = \gamma\tau_0. \tag{41}$$

If, in observation from S, the relativistic energy E of a body in S' becomes E/E_0 times the rest mass energy E_0, then the time elapsed in S' τ_0 becomes E_0/E times the time elapsed in S τ.

Now, let us consider an electron in an atom. If the solutions of Eqs. (11) and (12) are multiplied with Eq. (32), then the following equations are obtained:

$$E_{\mathrm{re}}L = E_0L_0, \tag{42}$$

$$m_{\mathrm{re}}L = m_{\mathrm{e}}L_0. \tag{43}$$

Also, if Eq. (32) is taken into account, these relations become

$$L = \frac{E_0}{E_{\mathrm{re}}}L_0 = \frac{m_{\mathrm{e}}}{m_{\mathrm{re}}}L_0 = \frac{L_0}{\gamma_{\mathrm{a}}}, \quad \gamma_{\mathrm{a}} = \frac{E_{\mathrm{re}}}{E_0} = \frac{m_{\mathrm{re}}}{m_{\mathrm{e}}} < 1. \tag{44}$$

Equation (44) signifies that when the relativistic energy E_{re} of an electron becomes E_{re}/E_0 times the rest mass energy E_0, length in the direction of motion of the electron in S' elongates by E_0/E_{re} (however, $E_{\mathrm{re}} < E_0$).

Incidentally, v is not included in Eq. (44). For this reason, Eq. (44) is thought to be an equation applicable when a particle is stationary. The aspect of a body, which contracts in STR, is length in the direction of motion. Therefore, if a particle is stationary, its length in the direction of motion cannot be discussed. It is likely valid to regard the L in Eq. (44) as the radius or diameter of the particle.

With regard to time, the following equations are obtained when Eqs. (11) and (12) are multiplied with Eq. (35):

$$E_{\mathrm{re}}\tau_0 = E_0\tau, \tag{45}$$

$$m_{\mathrm{re}}\tau_0 = m_{\mathrm{e}}\tau. \tag{46}$$

Also, if Eq. (35) is taken into account, these relations become

$$\tau = \frac{E_{\mathrm{re}}}{E_0}\tau_0 = \frac{m_{\mathrm{re}}}{m_{\mathrm{e}}}\tau_0 = \gamma_{\mathrm{a}}\tau_0. \tag{47}$$

If, in observation from S, the relativistic energy E_{re} of an electron becomes E_{re}/E_0 times the rest mass energy E_0, then the time elapsed in the electron system τ_0 becomes E_0/E_{re} times the time elapsed in S τ.

If motion in macrolevel free space is taken to be the problem, then there is no clear superior–inferior relationship between Eqs. (13) and (38) and Eqs. (14) and (41). However, if the discussion is extended to macrolevel space, then the situation changes. The length of a body moving at constant velocity v contracts in Eq. (13), but it can be predicted to elongate with Eq. (32).

If Eqs. (14) and (35) are taken into account with respect to time, then it is not the case that the tempo at which time elapses depends on the velocity of the coordinate system.

According to quantum mechanics, the velocity of an electron in a certain quantum mechanical state in a hydrogen atom is indeterminate and variable. The value of energy, in contrast, is determinate. Energy is a more intrinsic physical quantity than velocity.

If it is explained that L or τ is dependent on velocity, then the discussion must focus separately on macrolevel free space and microlevel space in the atom. However, if it is explained that L or τ depends on energy, then macro- and microlevel space can be discussed together.

IV. CONCLUSION

This paper has shown that L in Eq. (13) and τ in Eq. (14) predicted by STR can be expressed using the following equations if Eqs. (9) and (10) are taken into account:

$$L = \frac{E_0}{E}L_0 = \frac{m_0}{m}L_0 = \frac{L_0}{\gamma}, \quad \gamma = \frac{E}{E_0} = \frac{m}{m_0} > 1, \tag{48}$$

$$\tau = \frac{E}{E_0}\tau_0 = \frac{m}{m_0}\tau_0 = \gamma\tau_0. \tag{49}$$

On the other hand, if Eq. (27) can be applied in the space in the atom, then L and τ can be expressed using the following equations:

$$L = \frac{L_0}{\gamma_{\mathrm{a}}} = L_0\sqrt{1 + v^2/c^2}, \quad \gamma_{\mathrm{a}} = \frac{1}{\sqrt{1 + v^2/c^2}}, \tag{50}$$

$$\tau = \gamma_{\mathrm{a}}\tau_0 = \frac{\tau_0}{\sqrt{1 + v^2/c^2}}. \tag{51}$$

L and τ can also be expressed with the following equations if Eqs. (11) and (12) are taken into account:

$$L = \frac{E_0}{E_{\mathrm{re}}}L_0 = \frac{m_{\mathrm{e}}}{m_{\mathrm{re}}}L_0 = \frac{L_0}{\gamma_{\mathrm{a}}}, \quad \gamma_{\mathrm{a}} = \frac{E_{\mathrm{re}}}{E_0} = \frac{m_{\mathrm{re}}}{m_{\mathrm{e}}} < 1, \tag{52}$$

$$\tau = \frac{E_{\mathrm{re}}}{E_0}\tau_0 = \frac{m_{\mathrm{re}}}{m_{\mathrm{e}}}\tau_0 = \gamma_{\mathrm{a}}\tau_0. \tag{53}$$

In order to derive Eqs. (52) and (53), it is necessary to derive Eq. (27) beforehand. However, even without Eq. (27), it is possible to guess Eqs. (32) and (35) from Eqs. (13) and (14).

Therefore, it is possible to guess Eqs. (52) and (53), even without deriving Eq. (27). (The main purpose of this paper is not to derive Eq. (27). Hence, the validity of Eq. (27) is not verified in this paper.)

However, electrons are currently thought to be particles with no size. There is also no evidence that Eq. (27) holds in the space in the atom.

Therefore, the scope here is limited to STR equations applicable in macrolevel free space, but this paper concludes that Eqs. (48) and (49) are more profound than Eqs. (13) and (14).

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[1]K. Suto, Phys. Essays **24**, 301 (2011).
[2]L. I. Schiff, *Quantum Mechanics* (McGraw-Hill, New York, 1968), p. 486.
[3]K. Suto, Phys. Essays **24**, 598 (2011).
[4]K. Suto, Phys. Essays **25**, 488 (2012).
[5]A. P. French, *Special Relativity* (W.W. Norton & Co., New York, 1968), p. 78.
[6]K. Suto, Phys. Essays **22**, 135 (2009).
[7]A. P. French, *Special Relativity* (W.W. Norton & Co., New York, 1968), p. 97.
[8]A. P. French, *Special Relativity* (W.W. Norton & Co., New York, 1968), p. 100.