Chapter 7
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Dark Matter and Unknown Ultra-low Energy Levels of the Hydrogen Atom

Koshun Suto[1*]

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ABSTRACT

This paper discusses ultra-low energy levels of the hydrogen atom which was not predictable with quantum mechanics. The hydrogen atom has ultra-low energy levels. The existence of electrons at these energy levels can be demonstrated by changing the interpretation of triplet production. The matter formed from a proton with positive mass, and an electron with negative mass that orbits near that proton, is smaller than an ordinary hydrogen atom to an extreme degree. The radius is about 1.331×10^{-5} the radius of an ordinary hydrogen atom in the 1s state.
When this unknown matter gathers in large amounts, it becomes a huge mass. This paper identifies such matter as the true nature of dark matter, the mysterious matter that physicists are currently searching for.

Keywords: Dark matter; ultra-low energy levels; hydrogen atom; electron with negative energy.

1. INTRODUCTION

The universe is currently thought to contain matter whose true nature is unknown, in quantities far exceeding the known forms of matter such as atoms, molecules and celestial objects. Historically, the existence of this unknown matter was first pointed out by F. Zwicky in 1933.

In the latter half of the 1970s, it was ascertained, through highly precise observations by Vera Rubin, that an unknown source of gravity is present in galaxies. Today, the existence of this unknown mass (source of gravitational force) called "dark matter" (DM) is supported by many scientists. DM is thought to have the following characteristics [1].

(1) It is widely present in galactic systems.
(2) It is electrically neutral.
(3) It has considerable mass.
(4) It cannot be observed optically (it does not emit light).

DM candidates can be roughly divided into two types: elementary particle candidates and astrophysical candidates. The leading elementary particle candidate is a Weakly Interacting Massive Particles (WIMPs).

In addition, a new theory was announced which posits another force dominating the world of the extremely small, and the existence of a Strongly Interacting Massive Particle (SIMP) [2]. The simplest model assumes one type of particle is involved in DM. However, since the 1990s various experiments have attempted to directly detect WIMPs, but no definitive signs suggesting the existence of WIMPs have been found. Therefore, some scientists have doubts about the current theory, and have begun to also consider models of DM comprised of multiple particles [3,4].

This paper believes that DM does not necessary have to be elementary particle. The author published a paper predicting the existence of hydrogen atoms at ultra-low energy levels [5,6]. This paper examines whether such hydrogen atoms can be candidates for DM.

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[1]Independent Researcher, 5-24, Oote-Town, Isesaki, Japan.
**Corresponding author: E-mail: koshun_suto129@mbr.nifty.com;*

2. UNKNOWN ENERGY LEVELS OF THE HYDROGEN ATOM THAT CANNOT BE PREDICTED WITH QUANTUM MECHANICS

In 1913, Bohr derived a equation for the energy levels of the hydrogen atom by assuming quantum condition, and thereby explained the stability of the atom. Later, Dirac derived the relativistic wave equation, and obtained more precise solutions for energy levels which incorporates spin.

Incidentally, one of the most important relationships in the Special Theory of Relativity (STR) is as follows:

$$\left(mc^2\right)^2 = \left(m_0c^2\right)^2 + c^2p^2. \tag{1}$$

Here, mc^2 is the relativistic energy of an object or a particle, and m_0c^2 is the rest mass energy.

Currently, Einstein's relationship (1) is used to describe the energy and momentum of particles in free space, but for explaining the behavior of bound electrons inside atoms, opinion has shifted to quantum mechanics as represented by equations such as the Dirac's relativistic wave equation.

For reasons such as these, there was no search for a relationship between energy and momentum applicable to an electron in the hydrogen atom. However, the author has ventured to take up this problem, and derived the following relationship [7,8,9].

$$\left(m_nc^2\right)^2 = \left(m_\mathrm{e}c^2\right)^2 - c^2p_n^2, \quad m_n < m_\mathrm{e}. \tag{2}$$

Here, $m_\mathrm{e}c^2$ is the rest mass energy of the electron, and m_nc^2 is the relativistic energy of the electron. The subscript n is the principal quantum number.

From Eqs. (1) and (2) it is evident that, if a stationary electron begins to move in free space, or is incorporated into an atom, then the energy which serves as the departure point is the rest mass energy.

Consider the case where an electron currently stationary in free space is drawn to a proton to form a hydrogen atom. At this time, the rest mass energy of the electron decreases. We take this decrease in energy to be $-\Delta m_\mathrm{e}c^2$, the energy of the photon emitted by the electron to be $h\nu$, and the kinetic energy gained by the electron to be *K*.

For the law of conservation of energy to hold, the following relation must hold between these energies.

$$-\Delta m_\mathrm{e}c^2 + h\nu + K = 0. \tag{3}$$

The author presented the following equation as an equation indicating the relationship between the rest mass energy and potential energy of the electron in a hydrogen atom [10,11].

$$V\left(r\right) = -\Delta m_\mathrm{e}c^2. \tag{4}$$

According to this equation, the potential energy of a bound electron in a hydrogen atom is equal to the reduction in rest mass energy of that electron.

When describing the motion of a bound electron in a hydrogen atom, a term must be included in that equation for the potential energy. From this $E_{\mathrm{ab},n}$ can be defined as follows.

$$E_{\mathrm{ab},n} = m_nc^2 = m_\mathrm{e}c^2 + V\left(r_n\right) + K_n = m_\mathrm{e}c^2 - h\nu. \tag{5}$$

Here, m_n is the relativistic mass of the electron. The subscript n is the principal quantum number.

Incidentally, according to the virial theorem, the following relation holds between *K* and *V*:

$$\langle K\rangle = -\frac{1}{2}\langle V\rangle. \tag{6}$$

Here, *K* is the kinetic energy of the entire system, and *V* is the potential energy of the entire system.

The average time of *K* is equal to −1/2 the time average of *V*. Also, the sum of the time average *K* and the time average of the total mechanical energy *E* of the entire system becomes 0. That is,

$$\langle K\rangle + \langle E\rangle = 0. \tag{7}$$

Next, if Eqs. (6) and (7) are combined, the result is as follows:

$$\langle E\rangle = -\langle K\rangle = \frac{1}{2}\langle V\rangle. \tag{8}$$

Due to the above, $E_{\mathrm{ab},n}$ was defined classically as follows:

$$E_{\mathrm{ab},n} = m_{\mathrm{e}}c^2 + K_n + V(r_n) = m_{\mathrm{e}}c^2 - K_n = m_{\mathrm{e}}c^2 + E_n, \quad E_n < 0. \tag{9}$$

The following section considers unknown energy levels that can be derived from Eq. (2).

Potential energy is not incorporated into Eq. (2) in a form which is visible to the eye. However, a quantity corresponding to $V(r_n)$ is incorporated from the beginning into $E_{\mathrm{ab},n}$ due to the definition of Eq. (5).

There is a lower limit to potential energy, and the range which energy can assume is as follows.

$$-m_{\mathrm{e}}c^2 \le V(r) < 0. \tag{10}$$

Also, the following constraint holds regarding the relativistic energy $E_{\mathrm{ab},n}$ of the electron due to Eqs. (8) and (10) (here, the discussion is limited to the ordinary energy levels of the atom).

$$\frac{1}{2}m_{\mathrm{e}}c^2 \le E_{\mathrm{ab},n} < m_{\mathrm{e}}c^2. \tag{11}$$

Incidentally, it is known that the following formula can be derived from Eq. (1).

$$E = \pm m_0 c^2\left(1 - \frac{v^2}{c^2}\right)^{-1/2}. \tag{12}$$

If the same logic is applied to Eq. (2), then the following formula can be derived.

$$E_{\mathrm{ab},n} = \pm m_{\mathrm{e}}c^2\left(1 + \frac{v_n^2}{c^2}\right)^{-1/2}. \tag{13}$$

Here, *v* on the left side was set to v_n.An electron in a hydrogen atom becomes lighter in mass as it increases in speed. This is the opposite of the prediction of STR.

Next, Eq. (13) is rewritten using the following relationship derived in ref. [12].

$$\frac{v_n}{c} = \frac{\alpha}{n}. \tag{14}$$

When that is done,

$$m_n = \frac{m_e}{\left(1+\alpha^2/n^2\right)^{1/2}}, \quad m_n < m_e. \tag{15}$$

Here, α is the fine-structure constant, and is defined as follows.

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} = 7.2973525693\times10^{-3}. \tag{16}$$

When this is done, the following $E_{\mathrm{ab},n}$ and E_n can be derived from Eqs. (13) and (9) [13,14].

$$E_{\mathrm{ab},n}^{\pm} = \pm m_n c^2 \pm E_n = \pm m_e c^2\left(\frac{n^2}{n^2+\alpha^2}\right)^{1/2}, \qquad n = 0,1,2,\cdots. \tag{17}$$

$$E_n = m_e c^2\left[\left(\frac{n^2}{n^2+\alpha^2}\right)^{1/2} - 1\right]. \tag{18}$$

$E_{\mathrm{ab},n}$ of Eq. (17) define an absolute quantity, which includes the electron's rest mass energy.

Incidentally, the equation derived from classical quantum theory is following [15,16].

$$E_{\mathrm{BO},n} = -\frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{m_e e^4}{\hbar^2}\cdot\frac{1}{n^2} = -\frac{\alpha^2}{2n^2}m_e c^2, \quad n = 1,2,\cdots. \tag{19}$$

Here, the BO in $E_{\mathrm{BO},n}$ indicates the equation derived by Bohr.

Now, if a Taylor expansion is performed on the right side of Eq. (18),

$$E_n = m_e c^2\left[\left(1+\frac{\alpha^2}{n^2}\right)^{-1/2} - 1\right] \approx m_e c^2\left[\left(1-\frac{\alpha^2}{2n^2}+\frac{3\alpha^4}{8n^4}\right)-1\right] \approx m_e c^2\left(-\frac{\alpha^2}{2n^2}+\frac{3\alpha^4}{8n^4}\right). \tag{20}$$

When this is done, the equations for the energies is as follows.

$$E_n \approx -\frac{\alpha^2}{2n^2}m_e c^2. \tag{21}$$

From this, it is evident that Bohr's energy equation, Eq. (19), is an approximation of Eq. (18).

The following compares energies when $n = 1$.

Value predicated by this paper Eq. (18): $E_1 = -13.60515\ \mathrm{eV}$. (22a)

Value predicted by Bohr Eq. (19): $E_{\mathrm{BO},1} = -13.60569\ \mathrm{eV}$. (22b)

$$\frac{E_{\mathrm{BO},1}}{E_1} = 1.0000397. \tag{23}$$

Incidentally, there are also positive and negative solutions for $E_{\mathrm{ab},n}(m_n)$ in Eq. (2). Here, the ordinary, known energies of a hydrogen atom are expressed as $E^{+}_{\mathrm{ab},n}$, E^{+}_{n}. Also, the negative energies are expressed as $E^{-}_{\mathrm{ab},n}$, E^{-}_{n}.

The equations for positive energies is as follows.

$$E^{+}_{\mathrm{ab},n} = m_{\mathrm{e}}c^2\left(1+\frac{\alpha^2}{n^2}\right)^{-1/2} \approx m_{\mathrm{e}}c^2 - \frac{\alpha^2}{2n^2}m_{\mathrm{e}}c^2, \qquad n = 0,1,2,\cdots. \tag{24}$$

In contrast, the equations for the negative solutions are as follows.

$$E^{-}_{\mathrm{ab},n} = -m_{\mathrm{e}}c^2\left(1+\frac{\alpha^2}{n^2}\right)^{-1/2} \approx -m_{\mathrm{e}}c^2 + \frac{\alpha^2}{2n^2}m_{\mathrm{e}}c^2, \qquad n = 0,1,2,\cdots. \tag{25}$$

$$E^{-}_{n} \approx -2m_{\mathrm{e}}c^2 + \frac{\alpha^2}{2n^2}m_{\mathrm{e}}c^2. \tag{26}$$

If the above is indicated graphically, the result is as follows (Fig. 1).

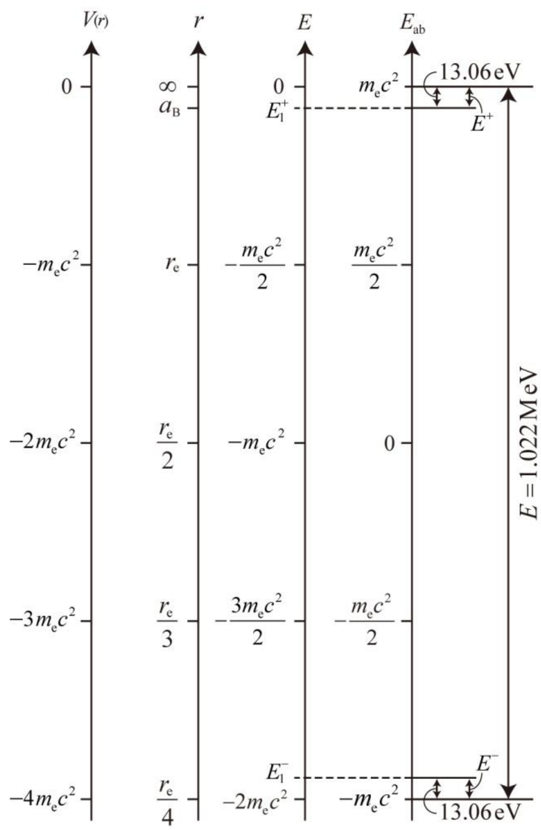

Fig. 1. The energy levels of the hydrogen atom predicted by classical quantum theory E^{+}_{n} ($E^{+}_{\mathrm{ab},n}$) and the new energy levels whose existence has been indicated by this paper E^{-}_{n} ($E^{-}_{\mathrm{ab},n}$)

If the relativistic mass of the electron is taken to be m_n , then the negative solution can be written as follows:

$$m_n^- = -m_e \left(1 + \frac{\alpha^2}{n^2}\right)^{-1/2} < 0. \tag{27}$$

3. PHENOMENA WHICH DEMONSTRATE THE EXISTENCE OF ULTRA-LOW ENERGY LEVELS

How can hydrogen atoms in this energy state be verified? This paper looks at triplet production.

It is generally assumed that in triplet production, in which 2 electrons and 1 positron are created, electron-pair creation occurs not near the atomic nucleus, but near the electron in the outer shell orbital. A total of three particles are created in this case: one outer shell electron forming the atom, and a positron and electron created through pair production. However, in this model, (1.022MeV- E_n^+) should be sufficient as the necessary photon energy. If an energy of 2.044MeV ($4m_ec^2$), is needed for triplet production, then the recoiled electron should be regarded as being at an ultra-low energy level.

Now, consider the case where an incident γ-ray has the energy corresponding to the mass of 4 electrons (2.044 MeV). If this is discussed classically, the γ-ray can create an electron and positron near $r = r_e / 2$ (Fig. 2).

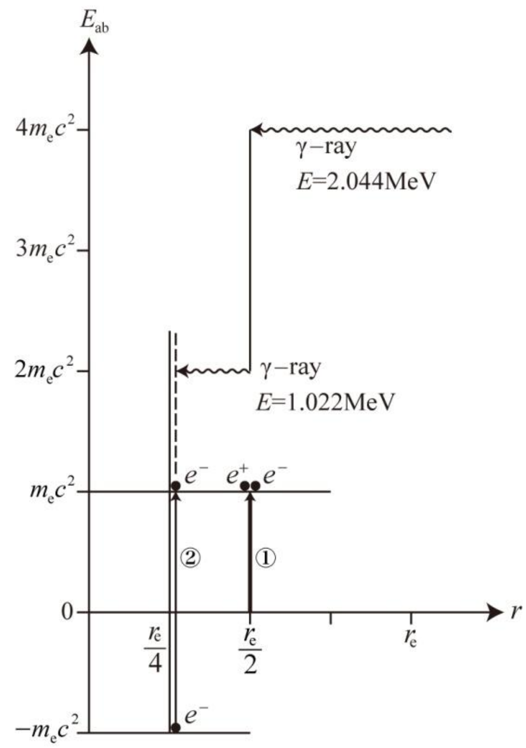

Fig. 2. Interpretation of this paper regarding triplet production

Consider the case where a γ-ray with the energy of 4 electrons (2.044 MeV) is incident on an atomic nucleus (proton).This γ-ray will give 1.022 MeV of energy to the virtual particles at $r = r_e / 2$, and an electron-positron pair will be created (↑①). When this γ-ray approaches closer to the atomic nuclear, and the electron in the orbital around the proton absorbs this energy, the electron will be excited and appear in free space (↑②). As a result, 2 electrons and 1 positron will appear in free space.

This paper points out that one of the two electrons which appears is an electron in the $E^{-}_{\mathrm{ab},n}$ state. A hydrogen atom in the $E^{-}_{\mathrm{ab},n}$ state will henceforth be called a “dark hydrogen atom” in this paper. Furthermore, the hydrogen molecule produced from ${}_{\mathrm{D}}\mathrm{H}$ will be called “dark hydrogen molecule”, and indicated as ${}_{\mathrm{D}}\mathrm{H}_2$.

This paper regards the existence of ${}_{\mathrm{D}}\mathrm{H}$ as demonstrated, albeit indirectly, by the fact the electrons forming ${}_{\mathrm{D}}\mathrm{H}$ have been observed.

4. CLASSICAL ORBITAL RADII OF THE ELECTRON AT THE UNKNOWN ENERGY LEVELS

This section discusses the unknown orbital radii of a hydrogen atom.

Incidentally, the energy of the hydrogen atom can also be written as follows.

$$E_n = -\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n} = -\frac{1}{2}\frac{r_\mathrm{e} m_\mathrm{e} c^2}{r_n} = -m_\mathrm{e}c^2\left(\frac{r_\mathrm{e}/2}{r_n}\right). \tag{28}$$

Here, r_e isthe classical electron radius as follows.

$$r_\mathrm{e} = \frac{e^2}{4\pi\varepsilon_0 m_\mathrm{e} c^2} = 2.8179403227\times 10^{-15}\ \mathrm{m}. \tag{29}$$

Also, the following equation for energy can be obtained from Eqs. (9) and (28).

$$E_{\mathrm{ab},n} = m_\mathrm{e}c^2 + E_n = m_\mathrm{e}c^2\left(1 - \frac{r_\mathrm{e}/2}{r_n}\right). \tag{30}$$

Here, if $-m_\mathrm{e}c^2$ is substituted for *E* in Eq. (28), then the *r* where $E_\mathrm{ab} = 0$ is:

$$r = \frac{r_\mathrm{e}}{2}. \tag{31}$$

The radius r where $E_\mathrm{ab} = 0$ is $r_\mathrm{e}/2$ due to Eq. (30). Dirac predicted that the vacuum energy E satisfies the relation $E < -m_\mathrm{e}c^2$, but actually $E_\mathrm{ab} = 0$ is the energy of the virtual electron-positron pair which make up the vacuum (Fig. 3).

In Dirac’s hole theory, when the γ-ray gives all of its energy to the virtual particles ($E = -2m_\mathrm{e}c^2$) comprising the vacuum around the atomic nucleus, a virtual particle acquires rest mass, and is emitted as an electron into free space, while the hole opened in the vacuum is the positron (Fig. 3a).

In the author’s interpretation, an electron-positron pair is created because a γ-ray with an energy of 1.022 MeV gives rest mass to a virtual electron-positron pair at the position $r = r_\mathrm{e}/2$ (Fig. 3b).

In quantum mechanics, *r* is an average value not a definitive value, and this paper follows that principle.

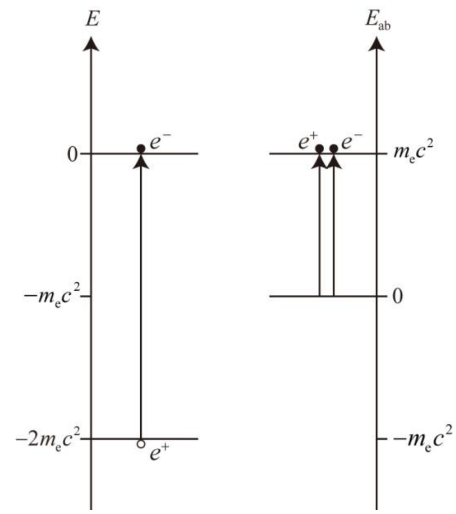

Fig. 3. Differences between Dirac's hole theory and the interpration in this paper

Dirac pointed out that there is a negative solution to Eq. (1) [17]. Adopting the same viewpoint, there is a negative solution to Eq. (2). To find the negative solution, it is necessary to create a quadratic equation for *r*. Thus,from Eqs. (17) and (30),

$$\frac{n^2}{n^2+\alpha^2}=\left(\frac{r_n-r_e/2}{r_n}\right)^2. \tag{32}$$

From this, the following quadratic equation is obtained.

$$r_n^2-\left(\frac{n^2+\alpha^2}{\alpha^2}\right)r_e r_n+\left(\frac{n^2+\alpha^2}{\alpha^2}\right)\frac{r_e^2}{4}=0. \tag{33}$$

If this equation is solved for r_n,

$$r_n^{\pm}=\frac{r_e}{2}\left(1+\frac{n^2}{\alpha^2}\right)\left[1\pm\left(1+\frac{\alpha^2}{n^2}\right)^{-1/2}\right]. \tag{34}$$

When the Taylor expansion of Eq. (34) is taken, the result is as follows.

$$r_n^{\pm}\approx\frac{r_e}{2}\left(1+\frac{n^2}{\alpha^2}\right)\left[1\pm\left(1-\frac{\alpha^2}{2n^2}+\frac{3\alpha^4}{8n^4}\right)\right]. \tag{35}$$

To begin, the positive solution is found first. (The positive solution is the solution found by Bohr.) The radii r_n^+ found from Eq. (35) are as follows.

$$r_n^+\approx\frac{3r_e}{4}+\frac{r_e}{\alpha^2}n^2=\frac{3r_e}{4}+a_B n^2. \tag{36}$$

Here, a_B is the Bohr radius as follows.

$$a_B=\frac{4\pi\varepsilon_0\hbar^2}{m_e e^2}=0.52917721067\times10^{-10}\text{ m}. \tag{37}$$

In contrast, the radii r_n found by Bohr are given by the following equation.

$$r_n = a_{\mathrm{B}} n^2, \ n = 1, 2, \cdots. \tag{38}$$

If Eqs. (36) and (38) are compared, it is evident that Eq. (38) is an approximation.

Next, the negative solution r_n^- of Eq. (35),

$$r_n^- \approx \frac{r_{\mathrm{e}}}{4} + \frac{\alpha^2 r_{\mathrm{e}}}{16n^2} = \frac{r_{\mathrm{e}}}{4} + \frac{a_{\mathrm{B}}}{n^2}\left(\frac{\alpha}{2}\right)^4. \tag{39}$$

Since r_n^- converges to $r_{\mathrm{e}}/4$, $r_{\mathrm{e}}/4$ can be regarded as the radius of the atomic nucleus of a hydrogen atom (i.e., the proton). Here, the theoretical value of the proton radius is:

$$\frac{r_{\mathrm{e}}}{4} = 0.704485080675 \times 10^{-15}\ \mathrm{m}. \tag{40}$$

Incidentally, the orbital radii r_n of the electron is a classical concept. In quantum mechanics, r_n is defined in each stationary state as the radii where the probability that the electron is present is maximal. In this paper too, r_n is used in the sense of quantum mechanics.

The next compares the orbital radii of an electron in a hydrogen atom r_n^+ and the orbital radii of an electron with a negative mass r_n^-. Referring to Eq. (34),

$$\frac{r_n^-}{r_n^+} = \left[1 - \left(\frac{n^2}{n^2 + \alpha^2}\right)^{1/2}\right]\left[1 + \left(\frac{n^2}{n^2 + \alpha^2}\right)^{1/2}\right]^{-1} = \frac{\left(n^2 + \alpha^2\right)^{1/2} - n}{\left(n^2 + \alpha^2\right)^{1/2} + n}. \tag{41}$$

Here, if we set $n = 1$,

$$\frac{r_1^-}{r_1^+} = \frac{\left(1 + \alpha^2\right)^{1/2} - 1}{\left(1 + \alpha^2\right)^{1/2} + 1} = 1.3312484168 \times 10^{-5}. \tag{42}$$

Next, if $\left(r_1^- - r_{\mathrm{e}}/4\right)$ and $r_{\mathrm{e}}/4$ are compared using r_1^- in Eq. (39).

$$\frac{r_1^- - r_{\mathrm{e}}/4}{r_{\mathrm{e}}/4} \approx \frac{\alpha^2}{4} = 1.33128 \times 10^{-5}. \tag{43}$$

From this, it is evident that the electron with negative mass is located near the atomic nucleus. Based on Eqs. (42) and (43), there is a possibility that the following relationship holds.

$$\frac{r_n^-}{r_n^+} = \frac{r_n^- - r_{\mathrm{e}}/4}{r_{\mathrm{e}}/4}. \tag{44}$$

Let's check this. Eq. (44) can be written as follows.

$$\frac{r_n^+ + r_n^-}{r_n^+} = \frac{r_n^-}{r_{\mathrm{e}}/4}. \tag{45}$$

From this,

$$\frac{r_{\mathrm{e}}}{4}\left(r_n^+ + r_n^-\right) = r_n^+ r_n^-. \tag{46}$$

First, from Eq. (34), the left side of Eq. (46) is,

$$\frac{r_{\mathrm{e}}}{4}\left(r_1^+ + r_n^-\right) = \left(\frac{r_{\mathrm{e}}}{2}\right)^2 \left(1 + \frac{n^2}{\alpha^2}\right). \tag{47}$$

Next, if Eq. (34) is used in the same way as the right side of Eq. (46),

$$r_n^+ r_n^- = \left(\frac{r_{\mathrm{e}}}{2}\right)^2 \left(1 + \frac{n^2}{\alpha^2}\right)^2 \left[1 - \left(\frac{n^2}{n^2 + \alpha^2}\right)\right] = \left(\frac{r_{\mathrm{e}}}{2}\right)^2 \left(1 + \frac{n^2}{\alpha^2}\right). \tag{48}$$

From Eqs. (47) and (48), it is evident that Eq. (44) holds. Also, the following equation can be derived from Eq. (44).

$$r_n^+ = \frac{r_{\mathrm{e}}}{4} \frac{r_n^-}{r_n^- - r_{\mathrm{e}}/4}. \tag{49}$$

$$r_n^- = \frac{r_{\mathrm{e}}}{4} \frac{r_n^+}{r_n^+ - r_{\mathrm{e}}/4}. \tag{50}$$

Also, the following relationship is obtained from Eqs. (41) and (44).

$$\frac{r_n^-}{r_n^+} = \frac{\left(n^2+\alpha^2\right)^{1/2} - n}{\left(n^2+\alpha^2\right)^{1/2} + n} = \frac{r_n^- - r_{\mathrm{e}}/4}{r_{\mathrm{e}}/4}. \tag{51}$$

When r_n^- is found from Eq. (51), the following two equations are obtained.

$$r_n^- = r_n^+ \cdot \frac{\left(n^2+\alpha^2\right)^{1/2} - n}{\left(n^2+\alpha^2\right)^{1/2} + n}. \tag{52}$$

$$r_n^- = \frac{r_{\mathrm{e}}}{2} \frac{\left(n^2+\alpha^2\right)^{1/2}}{\left(n^2+\alpha^2\right)^{1/2} + n}. \tag{53}$$

The following relationship holds due to Eqs. (50) and (52).

$$\frac{r_{\mathrm{e}}}{4} \frac{1}{r_n^+ - r_{\mathrm{e}}/4} = \frac{\left(n^2+\alpha^2\right)^{1/2} - n}{\left(n^2+\alpha^2\right)^{1/2} + n}. \tag{54}$$

When r_n^+ is found from Eq. (54),

$$r_n^+ = \frac{r_{\mathrm{e}}}{2} \frac{\left(n^2+\alpha^2\right)^{1/2}}{\left(n^2+\alpha^2\right)^{1/2} - n}. \tag{55}$$

Here, if we set $n=1$, Eq. (55) is,

$$r_1^+ = 0.52919834512 \times 10^{-10}\ \mathrm{m}. \tag{56}$$

Next, if r_1^+ and Bohr radius are compared,

$$\frac{r_1^+}{a_\mathrm{B}} = 1.0000399383. \tag{57}$$

Rearranging Eqs. (55) and (53), we obtain the following equation [18].

$$r_n^+ = \frac{r_\mathrm{e}}{2}\left[1 - \left(\frac{n^2}{n^2+\alpha^2}\right)^{1/2}\right]^{-1}, \qquad n = 0,1,2,\cdots. \tag{58}$$

$$r_n^- = \frac{r_\mathrm{e}}{2}\left[1 + \left(\frac{n^2}{n^2+\alpha^2}\right)^{1/2}\right]^{-1}, \qquad n = 0,1,2,\cdots. \tag{59}$$

Here, when $n=1$, Eq. (59) is,

$$r_1^- = 7.0449445912 \times 10^{-16}\ \mathrm{m}. \tag{60}$$

Next, if r_1^- and the proton radius are compared,

$$\frac{r_1^-}{r_\mathrm{e}/4} = 1.0000133124. \tag{61}$$

Also, if Eqs. (49) and (50) are combined,

$$\frac{r_n^-}{r_n^+} = \frac{r_\mathrm{e}/4}{r_n^+ - r_\mathrm{e}/4} = \frac{r_n^- - r_\mathrm{e}/4}{r_\mathrm{e}/4}. \tag{62}$$

Here, when $n=1$,Eq. (62) is,

$$\frac{r_1^-}{r_1^+} = \frac{r_1^- - r_\mathrm{e}/4}{r_\mathrm{e}/4} = \frac{\left(1+\alpha^2\right)^{1/2} - 1}{\left(1+\alpha^2\right)^{1/2} + 1} = 1.3312484168 \times 10^{-5} \approx \frac{1}{75120}. \tag{63}$$

From Eqs. (53) and (55), the following relationships hold between r_n^+ and r_n^-.

$$r_n^+ + r_n^- = r_\mathrm{e}\left(1 + \frac{n^2}{\alpha^2}\right). \tag{64}$$

$$r_n^+ - r_n^- = \frac{r_\mathrm{e}}{\alpha^2} n^2 \left(1 + \frac{\alpha^2}{n^2}\right)^{1/2} = a_\mathrm{B} n^2 \left(1 + \frac{\alpha^2}{n^2}\right)^{1/2} \approx a_\mathrm{B} n^2. \tag{65}$$

Incidentally, the author has previously derived the following equation.

$$\frac{E_{\mathrm{ab},n}}{m_\mathrm{e} c^2} = \frac{n}{\left(n^2+\alpha^2\right)^{1/2}} = \frac{m_n}{m_\mathrm{e}} = \frac{r_n - r_\mathrm{e}/2}{r_n}. \tag{66}$$

If Eqs. (65) and (66) are combined,

$$\frac{n}{\left(n^2+\alpha^2\right)^{1/2}}=\frac{m_n}{m_e}=\frac{r_n^+-r_e/2}{r_n^+}=\frac{a_B n^2}{r_n^+-r_n^-}. \tag{67}$$

The following relationship is also obtained from Eq. (62).

$$\left(r_n^+-\frac{r_e}{4}\right)\left(r_n^--\frac{r_e}{4}\right)=\left(\frac{r_e}{4}\right)^2. \tag{68}$$

Incidentally, there are vast clouds of hydrogen gas around the galaxy clusters that exist in space. The energy of matter made up of an electron with negative mass and a proton with positive mass is far less than the energy of an ordinary hydrogen atom. Naturally, this unknown matter exists in larger amounts than hydrogen gas. In addition, it is far smaller in size an ordinary hydrogen atom, and thus if these atoms gather in large amounts, they become a tremendous source of gravity. This matter is electrically neutral and stable. It is also extremely small in size, and therefore can pass through ordinary matter.

Thus, in this paper the system of an electron with negative mass and a proton with positive mass is regarded as the true nature of DM, the mysterious matter whose nature is unknown.

5. CONCLUSION

The hydrogen atom have energy levels far lower than the energy levels that can be derived from quantum mechanics. That is,

$$E_{ab,n}^-=-m_e c^2\left(\frac{n^2}{n^2+\alpha^2}\right)^{1/2}\approx -m_e c^2+\frac{\alpha^2}{2n^2}m_e c^2,\qquad n=0,1,2,\cdots. \tag{69}$$

The classical orbital radii of the electron at the energy levels in Eq. (69) is given by the following equation.

$$r_n^-=\frac{r_e}{4}\frac{r_n^+}{r_n^+-r_e/4}. \tag{70}$$

$$r_n^-=\frac{r_e}{2}\left[1+\left(\frac{n^2}{n^2+\alpha^2}\right)^{1/2}\right]^{-1},\qquad n=0,1,2,\cdots. \tag{71}$$

An electron at the energy levels in Eq. (69) exists near the atomic nucleus. The question of what to call this system of a proton and electron will be an issue for the future.

r_n^- is far smaller than r_n^+. That is,

$$\frac{r_1^-}{r_1^+}=1.3312484168\times10^{-5}\approx\frac{1}{75120}. \tag{72}$$

Matter with negative energy levels is formed from a proton with positive mass and an electron with negative mass. The author has given the name “dark hydrogen” to this matter. This matter is a strong candidate for dark matter, whose true nature is currently unknown.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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Biography of author(s)

Koshun Suto
Independent Researcher, 5-24, Oote-Town, Isesaki, Japan.

He was born in Japan, in January 1955, and majored in chemistry at university. he began studying the special theory of relativity (STR) around the time he went to university. In the first 1 or 2 years, he worked hard to understand the STR, but gradually he began to have doubts. Time passed, and he wrote a number of papers in the 1990s. This paper is the result of correcting and improving an unpublished manuscript he wrote in 1996. Around 2000, there was a debate on the correctness of the STR in the Japanese physical science magazine Parity. He joined this debate midway through, and pointed out the contradictions of the STR. That manuscript was published in the October 2001 issue of the magazine. His paper successfully refuting the STR was first published in a refereed journal in 2010. Also, in 2002, by taking Einstein's energy-momentum relationship as a hint, he has derived an energy-momentum relationship applicable inside a hydrogen atom where there is potential energy. After that, he was able to present a candidate for dark matter based on that relationship. He was invited as a speaker to the Applied Physics and Mathematics Conference in 2018 held in Tokyo in October 2018, and presented a candidate for dark matter. To date, he has published 32 papers in 8 journals outside Japan. He is currently working as a representative officer of a Buddhist temple.

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